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Superfield T-duality rules

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Abstract: A geometric treatment of T-duality as an operation which acts on differential forms in superspace allows us to derive the complete set of T-duality transformation rules which relate the superfield potentials of $D = 10$ type-IIB supergravity with those of type-IIA supergravity including Ramond-Ramond superfield potentials ($C^{(2n)}_{M_1\ldots M_{2n}}(Z)$ resp. $\hat{C}^{(2n+1)}_{M_1\ldots M_{2n+1}}(\hat{Z})$) and fermionic supervielbeins ($E_M^{\alpha 1,2}$ resp. $\hat{E}_M^{\alpha 1}, \hat{E}^{\,2}_{M\alpha}$). We show that these rules are consistent with the superspace supergravity constraints.

Keywords: M-Theory, String Duality, Supergravity Models, Superspaces.

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1. Introduction

T-duality is a perturbative symmetry of string theory which relates, for instance, the type-IIA and type-IIB superstring models (see, e.g., [1]). The study of the bosonic string action in a background admitting an isometry [2] provided an elegant representation of T-duality as a map between two spacetime field theories. Such a field theoretical representation of T-duality was studied for the bosonic limit of supergravity [3]–[11]. Progress on its supersymmetric generalization has been achieved only recently [12]–[14] (see [15, 16] for earlier study). In [12] the study of the T-duality map of the component-field expansion of the Green-Schwarz action for the type IIA superstring up to quadratic order in the fermionic coordinate functions $\hat{\theta}(\tau,\sigma)$ was used to derive the type-IIB superstring action with the same accuracy and, then, the model for 'massive type-IIA superstring' (superstring in the Roman's massive type-IIA supergravity background). The T-duality rules for gravitini have been found in [13]. Finally, in [14] the T-duality rules for NS-NS superfields [$E_M^a(Z)$, $B_{MN}(Z)$ and $\hat{E}_M^a(\hat{Z})$, $\hat{B}_{MN}(\hat{Z})$] and fermionic supervielbeins ($E_M^{\alpha 1,2}$ and $\hat{E}_M^{\alpha 1}, \hat{E}_{M\alpha}^{\ 2}$) were found by studying the relation between complete type-IIA and type-IIB superstring actions and their κ-symmetries. However, this approach did not allow to find the T-duality rules for Ramond-Ramond (RR) *superfield potentials* ($C^{(2n)}_{M_1\ldots M_{2n}}(Z)$ and $\hat{C}^{(2n+1)}_{M_1\ldots M_{2n+1}}(\hat{Z})$) and required significant efforts to extract the transformation rules for the components of the RR field strengths from Bianchi identities.

One of the messages of this paper is that the *complete* set of superspace T-duality rules (including the rules for the RR superfield potentials) can be obtained from the relation between the complete κ-symmetric actions for Dirichlet superbranes in type-IIA and type-IIB supergravity backgrounds and subsequent study of the exchange between the type-IIA and IIB superspace supergravity constraints. Namely, in the first stage, the comparison of the type-IIA super-D$(p+1)$-brane and type-IIB super-Dp-brane actions [17], which are known to be related by T-duality [1, 18], provides the T-duality transformation rules for all the *bosonic* superforms of type-IIB resp. IIA supergravity: bosonic supervielbeins ($E^a(Z)$ resp. $\hat{E}^a(\hat{Z})$), NS-NS superforms ($B_2(Z)$ resp. $\hat{B}_2(\hat{Z})$) and all RR superforms ($C_{2n}(Z)$ resp. $\hat{C}_{2n\pm 1}(\hat{Z})$). Then, in a second stage, substituting these rules into the superspace torsion constraints and the constraints on NS-NS field strengths of type-IIA and type-IIB supergravities [19, 20, 17], one can derive the T-duality rules for the remaining (fermionic) supervielbein forms.

It turns out that the T-duality transformation rules for the bosonic superforms, which can be obtained from the comparison of the super-Dp-brane actions (i.e. by the superfield generalization of the *method* of ref. [11]), can be reproduced as well by a straightforward superfield (superform) generalization of the *final results* of ref. [11].[1] In this paper we shall use such a shortcut, it will allow us, in particular, to simplify notations. By substituting

[1] Such a simple possibility to reproduce the superfield results from the component ones can be regarded as a reflection of the existence of the 'rheonomic' (group manifold) approach to supergravity [21] (see [22] for its superbrane generalization) which allows to lift the component equations (written in terms of differential forms on spacetime) to the superspace equations for superforms. This is also natural in a view of recent observation [23] that superfield description of the dynamical supergravity-superbrane interacting system (still hypothetical for $D=10,11$) is gauge equivalent to a more simple dynamical system described by

then the NS-NS T-duality rules thus obtained into the superspace torsion constraints and into the constraints on NS-NS field strengths [19, 20, 17], we derive the T-duality rules for fermionic supervielbein forms in Einstein frame.[2] Finally, we describe the verification of the consistency of the complete set of T-duality rules thus obtained with the superspace constraints for RR superform field strengths [17]. The T-duality transformation rules are collected in an appendix.

2. Basic notions and notations

The tangent space metric is mostly minus, $\eta_{\hat{a}\hat{b}} = \mathrm{diag}(+1,-1,\ldots,-1) = \eta_{ab}$. The hat symbol $\hat{}$ is used to distinguish superfields, coordinates and indices of the type-IIA supergravity; the superfields and coordinates of type-IIB superspace are denoted by the same symbols, but without hat. Bosonic supervielbein forms of type-IIA and type-IIB supergravity are denoted by

$$\text{type IIA}: \quad \hat{E}^{\hat{a}} = d\hat{Z}^{\hat{M}} \hat{E}^{\hat{a}}_{\hat{M}}(\hat{Z})\,, \tag{2.1}$$

$$\text{type IIB}: \quad E^{a} = dZ^{M} E^{a}_{M}(Z)\,, \tag{2.2}$$

NS-NS gauge *superforms* by

$$\text{type IIA}: \ \hat{B}_2 = \frac{1}{2} d\hat{Z}^{\hat{N}} \wedge d\hat{Z}^{\hat{M}} \hat{B}_{\hat{M}\hat{N}}(\hat{Z}) = \frac{1}{2}\hat{E}^{\hat{B}} \wedge \hat{E}^{\hat{A}} \hat{B}_{\hat{A}\hat{B}}(\hat{Z})\,, \tag{2.3}$$

$$\text{type IIB}: \ B_2 = \frac{1}{2} dZ^{N} \wedge dZ^{M} B_{MN}(Z) = \frac{1}{2}E^{B} \wedge E^{A} B_{AB}(Z)\,, \tag{2.4}$$

and the fermionic supervielbein forms by

$$\text{type IIA}: \hat{E}^{\hat{\alpha}} = (\hat{E}^{\alpha 1}, \hat{E}^{2}_{\alpha})\,, \quad \hat{E}^{\alpha 1} = d\hat{Z}^{\hat{M}} \hat{E}^{\alpha 1}_{\hat{M}}(\hat{Z})\,, \quad \hat{E}^{2}_{\alpha} = d\hat{Z}^{\hat{M}} \hat{E}_{\hat{M}\alpha}{}^{2}(\hat{Z})\,, \tag{2.5}$$

$$\text{type IIB}: E^{\check{\alpha}} = (E^{\alpha 1}, E^{\alpha 2})\,, \quad E^{\alpha 1} = dZ^{M} E^{\alpha 1}_{M}(Z)\,, \quad E^{\alpha 2} = dZ^{M} E^{\alpha 2}_{M}(Z)\,. \tag{2.6}$$

Here $\alpha = 1,\ldots,16$ is $D = 10$ Majorana-Weyl spinor index. Upper and lower indices correspond to opposite chiralities. Ten dimensional 16×16 sigma matrices, $\sigma^{a}_{\alpha\beta}$, $\tilde{\sigma}^{a\alpha\beta}$ are real, symmetric and satisfy $(\sigma^a\tilde{\sigma}^b + \sigma^b\tilde{\sigma}^a)_\alpha{}^\beta = 2\eta^{ab}\delta_\alpha{}^\beta$; $\sigma^{ab} = \sigma^{[a}\tilde{\sigma}^{b]} = 1/2(\sigma^a\tilde{\sigma}^b - \sigma^b\tilde{\sigma}^a)$, $\tilde{\sigma}^{ab} = \tilde{\sigma}^{[a}\sigma^{b]}$, $\sigma^{abc} = \sigma^{[a}\tilde{\sigma}^{b}\sigma^{c]}$, $\tilde{\sigma}^{abc} = \tilde{\sigma}^{[a}\sigma^{b}\tilde{\sigma}^{c]}$, etc. Finally,

$$\text{type IIA}: \quad \hat{C} = \hat{C}_1 \oplus \hat{C}_3 \oplus \hat{C}_5 \oplus \hat{C}_7 \oplus \hat{C}_9\,, \tag{2.7}$$

$$\text{type IIB}: \quad C = C_0 \oplus C_2 \oplus C_4 \oplus C_6 \oplus C_8 \oplus C_{10} \tag{2.8}$$

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the sum of the standard (component) supergravity action and the action for pure bosonic brane (the pure bosonic limit of the original superbrane action).

[2] Note that, although the authors of [14] worked in the so-called string frame, where the superstring action does not include the dilaton superfield, one can verify that the requirement of superstring κ-symmetry in the presence of standard type-II supergravity constraints [17] (see eqs. (5.1)–(5.4) below, or, equivalently, ref. [14]: eqs. (A.8), (A.9), (A.19)) would result in the trivialization of supergravity background *if* one were to drop the dilaton factor from the superstring action. This indicates that the standard superfield supergravity constraints have been formulated in the so-called Einstein frame rather than in the string frame. Hence, the superfield T-duality rules in the Einstein frame shall be more accessible and more useful for applications.

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denote the formal sums of all type IIA (odd) and all type IIB (even) RR superforms

$$\text{type IIA}: \quad \hat{C}_{2n+1} = \frac{1}{(2n+1)!} d\hat{Z}^{\hat{M}_{2n+1}} \wedge \cdots \wedge d\hat{Z}^{\hat{M}_1} \hat{C}^{(2n+1)}_{\hat{M}_1 \ldots \hat{M}_{2n+1}}(\hat{Z}) , \tag{2.9}$$

$$\text{type IIB}: \quad C_{2n} = \frac{1}{2n!} dZ^{M_{2n}} \wedge \ldots \wedge dZ^{M_1} C^{(2n)}_{M_1 \cdots M_{2n}}(Z) . \tag{2.10}$$

2.1 Isometries and underlying superspace $\mathcal{M}^{(11|32)}$

The coordinates associated with the isometry directions of type-IIA and type-IIB superspaces are denoted by $\hat{z}$ and y, respectively. The existence of such isometries provides the necessary condition for the existence of a T-duality map. Then we identify *all* the remaining superspace coordinates of *curved* type IIA and type-IIB superspace, i.e.

$$\text{type IIA}: \quad \mathcal{M}^{(10|32)}_{IIA} \; : \quad \hat{Z}^{\hat{M}} = (\tilde{Z}^{\tilde{M}}, \hat{z}) , \tag{2.11}$$

$$\text{type IIB}: \quad \mathcal{M}^{(10|32)}_{IIB} \; : \quad Z^{M} = (\tilde{Z}^{\tilde{M}}, y) . \tag{2.12}$$

In other words, we assume that the intersection of curved type-IIA and type-IIB superspaces, $\mathcal{M}^{(10|32)}_{IIA}$ and $\mathcal{M}^{(10|32)}_{IIB}$, defines some $D=9$, $N=2$ superspace $\mathcal{M}^{(9|32)}$

$$\mathcal{M}^{(10|32)}_{IIA} \cap \mathcal{M}^{(10|32)}_{IIB} = \mathcal{M}^{(9|32)} \tag{2.13}$$

$$\mathcal{M}^{(9|32)} \; : \tilde{Z}^{\tilde{M}} \equiv \left(\tilde{X}^{\tilde{m}}, \theta^{\mu}\right) , \qquad \tilde{m} = 0, \ldots, 8 , \qquad \mu = 1, \ldots, 32 . \tag{2.14}$$

This implies that *we consider T-duality as an operation acting on differential forms in superspace rather than on the superspace coordinates.* Such a possibility is guarantied by (super)diffeomorphism invariance of (superspace super)gravity, i.e. by its gauge symmetry under arbitrary changes of local coordinate system (in superspace).[3] However, such 'picture changing' allows to breakthrough the problems which hampered the way to superfield T-duality rules (see e.g. [25]).

Moreover, this point of view makes transparent that type-IIA and type IIB theories with isometries $\partial_{\hat{z}}$ and ∂_y can be defined on the hypersurfaces $\hat{z} = 0$ and $y = 0$ of an *underlying superspace* $\mathcal{M}^{(11|32)}$ with 11 bosonic and 32 fermionic coordinates,

$$\mathcal{M}^{(11|32)} \; : \quad (\tilde{Z}^{\tilde{M}}, \; y, \; \hat{z} \;) . \tag{2.15}$$

The notion of the underlying superspace $\mathcal{M}^{(11|32)}$ will be useful to obtain the superfield T-duality rules. An unholonomic basis of $\mathcal{M}^{(11|32)}$ should contain 11 bosonic superforms which could be chosen as 'mostly IIA',

$$(\hat{E}^{\hat{a}}, E^{*}) \equiv (\hat{E}^{\tilde{a}}, \hat{E}^{\#}, E^{*}) , \tag{2.16}$$

or as 'mostly IIB',

$$(E^{a}, \hat{E}^{\#}) \equiv (E^{\tilde{a}}, E^{*}, \hat{E}^{\#}) . \tag{2.17}$$

This basis should also contain 32 fermionic supervielbein forms, as the underlying superspace has 32 fermionic directions. It is convenient to use either IIA forms (2.5) or IIB forms (2.6).

[3] (Super)diffeomorphism invariance allows one to replace any coordinate transformations by the equivalent transformations of the supergravity (super)fields (see, e.g., [24] and refs. therein).

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2.2 On chirality and fermionic coordinates.

The possibility of identification of the fermionic coordinates of *curved* type-IIA and type-IIB superspaces, $\hat{\theta}^{\hat{\mu}} = \theta^{\mu}$, used before eqs. (2.11), (2.12), would not seem surprising if one remembers that the fermionic coordinates of a general *curved* superspace do not carry any chirality. In contradistinction to the case of flat superspace, the indices $\hat{\mu}$ and μ are *not* the spinor indices of the Lorentz group; $\hat{\theta}^{\hat{\mu}}$ and θ^{μ} are rather transformed by the general superdiffeomorphism symmetry. On the other hand, the *chirality*, which is used to distinguish the type-IIA and type-IIB cases, is defined through the projectors constructed from $D = 10$ Dirac matrices and, thus, is related to the concept of SO$(1,9)$ Lorentz group spinor representation. Hence, in curved superspace, *the chirality is a characteristic of the fermionic supervielbein 1-forms*, eqs. (2.5), (2.6), which do carry SO$(1,9)$ spinor indices.

In both type IIA and type IIB they can be considered as a pair of fermionic forms carrying Majorana-Weyl SO$(1,9)$ spinor indices $\alpha = 1, \ldots, 16$. As there is no charge conjugation matrix in the Majorana-Weyl spinor representation of SO$(1,9)$, there is no way to lower or to raise the spinor indices. Thus the chirality can be identified with the position of the spinor indices of the fermionic supervielbein forms. The type-IIB theory has both fermionic supervielbeins of the same chirality (2.6) and is chiral, while the type-IIA theory has fermionic supervielbein forms of opposite chiralities (2.5) and is nonchiral.

However, this does not imply different properties of the fermionic coordinates of the curved type-IIA and type-IIB superspaces. Only in the flat superspace limits, when one takes the fermionic supervielbein to be derivatives of the fermionic coordinates, the chiral structure, together with the definite spinor representation of the Lorentz group, becomes adjusted to the fermionic coordinates of flat superspace.

2.3 Isometries and differential forms in superspace

Arbitrary type-IIA (type-IIB) super-q-forms

$$\begin{aligned} \hat{\Omega}_q &= \frac{1}{q!} d\hat{Z}^{M_q} \wedge \cdots \wedge d\hat{Z}^{M_1} \hat{\Omega}_{M_1 \ldots M_q}(\hat{Z}) \,, \\ \Omega_q &= \frac{1}{q!} dZ^{M_q} \wedge \cdots \wedge dZ^{M_1} \Omega_{M_1 \ldots M_q}(Z) \end{aligned} \tag{2.18}$$

allow the decomposition

$$\hat{\Omega}_q = \hat{\Omega}_q^{(-)} + i_{\hat{z}} \hat{\Omega} \wedge d\hat{z} \,, \qquad \Omega_q = \Omega_q^{(-)} + i_y \hat{\Omega} \wedge dy \,, \tag{2.19}$$

into parts which, respectively, contain and do not contain the differentials of the isometry coordinate, $d\hat{z}$ or dy,

$$i_{\hat{z}} \hat{\Omega}_q := \frac{1}{(q-1)!} d\tilde{Z}^{\tilde{M}_{q-1}} \wedge \cdots \wedge d\tilde{Z}^{\tilde{M}_1} \hat{\Omega}_{\hat{z}\tilde{M}_1 \ldots \tilde{M}_{q-1}}(\tilde{Z}) \,, \tag{2.20}$$

$$\hat{\Omega}_q^{(-)} := \frac{1}{q!} d\tilde{Z}^{\tilde{M}_q} \wedge \cdots \wedge d\tilde{Z}^{\tilde{M}_1} \hat{\Omega}_{\tilde{M}_1 \ldots \tilde{M}_q}(\tilde{Z}) \,; \tag{2.21}$$

$$i_y \hat{\Omega} := \frac{1}{(q-1)!} d\tilde{Z}^{\tilde{M}_{q-1}} \wedge \cdots \wedge d\tilde{Z}^{\tilde{M}_1} \Omega_{y\tilde{M}_1 \ldots \tilde{M}_{q-1}}(\tilde{Z}) \,, \tag{2.22}$$

$$\Omega_q^{(-)} := \frac{1}{q!} d\tilde{Z}^{\tilde{M}_q} \wedge \cdots \wedge d\tilde{Z}^{\tilde{M}_1} \Omega_{\tilde{M}_1 \ldots \tilde{M}_q}(\tilde{Z}) \,. \tag{2.23}$$

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The conditions of isometry, that is of independence of all superfields on the coordinate $\hat{z}$ resp. y, have been already indicated in eqs. (2.20)–(2.23).

We find convenient to use the supervielbein forms adapted to the isometry, i.e. to assume that they obey the superfield generalization of the Kaluza-Klein ansatz [26]. For the bosonic supervielbein forms it implies the separation of the one tangent (super)space bosonic direction, denoted by $\hat{a} = \#$ for type IIA and $a = *$ for type IIB,

$$\begin{aligned} \text{type IIA} : \hat{E}^{\hat{a}} &= (\hat{E}^{\tilde{a}}, \hat{E}^{\#}) \,, & (2.24) \\ \text{type IIB} : E^{a} &= (E^{\tilde{a}}, E^{*}) \,, & (2.25) \\ \tilde{a} &= 0, 1, \ldots, 8 \,, \end{aligned}$$

and the assumption that the subsets of nine bosonic one-forms $\hat{E}^{\tilde{a}}$ and $E^{\tilde{a}}$ are defined on the nine-dimensional superspace $\mathcal{M}^{(9|32)}$ (2.13)

$$\begin{aligned} \text{type IIA} : \hat{E}^{\tilde{a}} &= \hat{E}^{\tilde{a}(-)} = d\tilde{Z}^{\tilde{M}} \hat{E}^{\tilde{a}}_{\tilde{M}}(\tilde{Z}) \qquad \Rightarrow \qquad i_{\hat{z}} \hat{E}^{\tilde{a}} \equiv \hat{E}^{\tilde{a}}_{\hat{z}} = 0 \,, & (2.26) \\ \text{type IIB} : E^{\tilde{a}} &= E^{\tilde{a}(-)} = d\tilde{Z}^{\tilde{M}} E^{\tilde{a}}_{\tilde{M}}(\tilde{Z}) \qquad \Rightarrow \qquad i_{y} E^{\tilde{a}} \equiv E^{\tilde{a}}_{y} = 0 \,. & (2.27) \end{aligned}$$

We find convenient to use the notation $i_{\hat{z}}\hat{E}^{\tilde{a}}$ and $i_y E^{\tilde{a}}$ instead of $\hat{E}^{\tilde{a}}_{\hat{z}}$ and $E^{\tilde{a}}_{y}$.

Thus $d\hat{z}$ and dy differentials appear only in the bosonic superforms $\hat{E}^{\#}$ and E^{*} respectively,

$$\begin{aligned} \text{type IIA} :\; & \hat{E}^{\#} = \hat{E}^{\#(-)} + i_{\hat{z}}\hat{E}^{\#} d\hat{z} \,, \\ & \hat{E}^{\#(-)} = d\tilde{Z}^{\tilde{M}} \hat{E}^{\#}_{\tilde{M}}(\tilde{Z}) \,, \qquad i_{\hat{z}}\hat{E}^{\#} = \hat{E}^{\#}_{\hat{z}}(\tilde{Z}) \,, & (2.28) \\ \text{type IIB} :\; & E^{*} = E^{*(-)} + i_y E^{*} dy \,, \\ & E^{*(-)} = d\tilde{Z}^{\tilde{M}} E^{*}_{\tilde{M}}(\tilde{Z}) \,, \qquad i_y E^{*} = E^{*}_{y}(\tilde{Z}) \,, & (2.29) \end{aligned}$$

but all the superfields in the decompositions (2.28), (2.29) depend only on the coordinates $\tilde{Z}^{\tilde{M}}$ of the nine-dimensional superspace (2.13).

The superfield generalization of the Kaluza-Klein ansatz [26] allows the appearance of $d\hat{z}$ (dy) terms in the fermionic supervielbein forms (2.5), (2.6) as well. For superspace calculations it is convenient to redefine the decomposition (2.19) of the fermionic forms by using the bosonic supervielbein forms $\hat{E}^{\#}$ and E^{*} instead of $d\hat{z}$ (dy),

$$\begin{aligned} \text{type IIA} : \hat{E}^{\alpha 1} &= \hat{E}^{\alpha 1(-)} + i_{\hat{z}}\hat{E}^{\alpha 1} d\hat{z} = \hat{E}^{\alpha 1[-]} + \hat{E}^{\#} \frac{i_{\hat{z}}\hat{E}^{\alpha 1}}{i_{\hat{z}}\hat{E}^{\#}} \,, \\ \hat{E}^{2}_{\alpha} &= \hat{E}^{2(-)}_{\alpha} + i_{\hat{z}}\hat{E}^{2}_{\alpha} d\hat{z} = \hat{E}^{2[-]}_{\alpha} + \hat{E}^{\#} \frac{i_{\hat{z}}\hat{E}^{2}_{\alpha}}{i_{\hat{z}}\hat{E}^{\#}} \,, & (2.30) \\ \text{type IIB} : E^{\alpha 1} &= E^{\alpha 1(-)} + i_y E^{\alpha 1} dy = E^{\alpha 1[-]} + E^{*} \frac{i_y E^{\alpha 1}}{i_y E^{*}} \,, \\ E^{\alpha 2} &= E^{\alpha 2(-)} + i_y E^{\alpha 2} dy = E^{\alpha 2[-]} + E^{*} \frac{i_y E^{\alpha 2}}{i_y E^{*}} \,. & (2.31) \end{aligned}$$

Clearly, the relations between the forms $\hat{E}^{\alpha 1[-]}$, $\hat{E}^{2[-]}_{\alpha}$, $E^{\alpha 1,2[-]}$ and the forms $\hat{E}^{\alpha 1(-)}$, $\hat{E}^{2(-)}_{\alpha}$, $E^{\alpha 1,2(-)}$ of the standard decomposition (2.20)–(2.23) read

$$\text{type IIA} : \hat{E}^{\alpha 1(-)} = \hat{E}^{\alpha 1[-]} + \hat{E}^{\#(-)} \frac{i_{\hat{z}} \hat{E}^{\alpha 1}}{i_{\hat{z}} \hat{E}^{\#}}\,, \qquad \hat{E}^{2(-)}_{\alpha} = \hat{E}^{2[-]}_{\alpha} + \hat{E}^{\#(-)} \frac{i_{\hat{z}} \hat{E}^{2}_{\alpha}}{i_{\hat{z}} \hat{E}^{\#}}\,, \tag{2.32}$$

$$\text{type IIB} : E^{\alpha 1(-)} = E^{\alpha 1[-]} + E^{*(-)} \frac{i_y E^{\alpha 1}}{i_y E^{*}}\,, \qquad E^{\alpha 2(-)} = E^{\alpha 2[-]} + E^{*(-)} \frac{i_y E^{\alpha 2}}{i_y E^{*}}\,. \tag{2.33}$$

Such a decomposition is useful as well for the spin connections,

$$\hat{w}^{\hat{a}\hat{b}} = \hat{w}^{\hat{a}\hat{b}(-)} + d\hat{z}\hat{w}^{\hat{a}\hat{b}}_{\hat{z}}(\tilde{Z}) = \hat{w}^{\hat{a}\hat{b}[-]} + \frac{\hat{E}^{\#}}{i_{\hat{z}}\hat{E}^{\#}} \hat{w}^{\hat{a}\hat{b}}_{\hat{z}}(\tilde{Z})\,, \tag{2.34}$$

$$\hat{w}^{\hat{a}\hat{b}[-]} = \hat{w}^{\hat{a}\hat{b}(-)} - \frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}} \hat{w}^{\hat{a}\hat{b}}_{\hat{z}}(\tilde{Z}) \tag{2.35}$$

$$\equiv \hat{E}^{\tilde{a}(-)} \hat{w}^{\hat{a}\hat{b}}_{\tilde{a}}(\tilde{Z}) + \hat{E}^{\alpha 1[-]} \hat{w}^{\hat{a}\hat{b}}_{\alpha 1}(\tilde{Z}) + \hat{E}^{2[-]}_{\alpha} \hat{w}_2^{\alpha \hat{a}\hat{b}}(\tilde{Z})\,, \tag{2.36}$$

$$w^{ab} = w^{ab(-)} + dy w^{ab}_y(\tilde{Z}) = w^{ab[-]} + \frac{E^{*}}{i_y E^{*}} w^{ab}_y(\tilde{Z})\,, \tag{2.37}$$

$$w^{ab[-]} = w^{ab(-)} - \frac{E^{*}}{i_y E^{*}} w^{ab}_y(\tilde{Z}) = E^{\tilde{a}(-)} w^{ab}_{\tilde{a}}(\tilde{Z}) + E^{\alpha 1[-]} w^{ab}_{\alpha 1}(\tilde{Z}) + E^{\alpha 2[-]} w^{ab}_{\alpha 2}(\tilde{Z})\,. \tag{2.38}$$

The use of supervielbein forms adapted to the isometry allows us to write the superfield T-duality rules in a compact form.

3. Résumé of bosonic T-duality rules

Let us begin by a brief résumé of the well-known bosonic results. The T-duality rules for NS-NS fields (Buscher rules) [2] have the simplest form in the string frame

$$\text{String frame} : g^{(s)}_{yy} = \frac{1}{\hat{g}^{(s)}_{\hat{z}\hat{z}}}\,, \qquad g^{(s)}_{\tilde{m}y} = \frac{1}{\hat{g}_{\hat{z}\hat{z}}} \hat{B}_{\hat{z}\tilde{m}}\,, \tag{3.1}$$

$$g^{(s)}_{\tilde{m}\tilde{n}} = \hat{g}^{(s)}_{\tilde{m}\tilde{n}} + \frac{1}{\hat{g}^{(s)}_{\hat{z}\hat{z}}} \left(\hat{B}_{\tilde{m}\hat{z}} \hat{B}_{\tilde{n}\hat{z}} - \hat{g}^{(s)}_{\tilde{m}\hat{z}} \hat{g}^{(s)}_{\tilde{n}\hat{z}} \right)\,. \tag{3.2}$$

$$e^{2\Phi} = -\frac{e^{2\hat{\Phi}}}{\hat{g}^{(s)}_{\hat{z}\hat{z}}}\,, \tag{3.3}$$

$$B_{\tilde{m}\tilde{n}} = \hat{B}_{\tilde{m}\tilde{n}} + \frac{1}{\hat{g}^{(s)}_{\hat{z}\hat{z}}} \left(\hat{g}^{(s)}_{\tilde{m}\hat{z}} \hat{B}_{\tilde{n}\hat{z}} - \hat{g}^{(s)}_{\tilde{n}\hat{z}} \hat{B}_{\tilde{m}\hat{z}} \right)\,,$$

$$B_{y\tilde{m}} = \frac{1}{\hat{g}^{(s)}_{\hat{z}\hat{z}}} \hat{g}^{(s)}_{\tilde{m}\hat{z}}\,. \tag{3.4}$$

In [11] they were rederived from the relation between type-IIA and type-IIB D-brane actions in purely bosonic supergravity background (i.e. in the background of the bosonic fields of the supergravity multiplets). Moreover, in [11] the rules for the RR gauge fields were obtained as well. They are

$$C^{(0)} = \hat{C}^{(1)}_{\hat{z}}\,, \tag{3.5}$$

$$C^{(2n)}_{y\tilde{m}_1\ldots\tilde{m}_{2n-1}} = \hat{C}^{(2n-1)}_{\tilde{m}_1\ldots\tilde{m}_{2n-1}} + (2n-1)\frac{\hat{C}^{(2n-1)}_{\hat{z}[\tilde{m}_1\ldots\tilde{m}_{2n-2}}\hat{g}_{\tilde{m}_{2n-1}]\hat{z}}}{\hat{g}_{\hat{z}\hat{z}}}\,, \tag{3.6}$$

$$\begin{aligned} C^{(2n)}_{\tilde{m}_1\ldots\tilde{m}_{2n}} &= \hat{C}^{(2n+1)}_{\hat{z}\tilde{m}_1\ldots\tilde{m}_{2n}} + 2n\hat{C}^{(2n-1)}_{[\tilde{m}_1\ldots\tilde{m}_{2n-1}}\hat{B}_{\tilde{m}_{2n}]\hat{z}} + \\ &\quad + 2n(2n-1)\frac{\hat{C}^{(2n-1)}_{\hat{z}[\tilde{m}_1\ldots\tilde{m}_{2n-2}}\hat{B}_{\hat{z}\tilde{m}_{2n-1}}\hat{g}_{\tilde{m}_{2n}]\hat{z}}}{\hat{g}_{\hat{z}\hat{z}}}\,, \end{aligned} \tag{3.7}$$

$$\text{for } n = 1,2,3,4\,,$$

$$C^{(10)}_{y\tilde{m}_1\ldots\tilde{m}_9} = \hat{C}^{(9)}_{\tilde{m}_1\ldots\tilde{m}_9} + 9\frac{\hat{C}^{(9)}_{\hat{z}[\tilde{m}_1\ldots\tilde{m}_8}\hat{g}_{\tilde{m}_9]\hat{z}}}{\hat{g}_{\hat{z}\hat{z}}}\,. \tag{3.8}$$

In the Einstein frame, where the metric is redefined as

$$\hat{g}^{(s)}_{\hat{m}\hat{n}} = e^{\frac{\hat{\Phi}}{2}}\hat{g}_{\hat{m}\hat{n}}\,, \qquad g^{(s)}_{mn} = e^{\frac{\Phi}{2}}g_{mn}\,, \tag{3.9}$$

the NS-NS T-duality rules read

$$e^{\frac{\Phi}{2}}g_{yy} = \frac{1}{e^{\frac{\hat{\Phi}}{2}}\hat{g}_{\hat{z}\hat{z}}}\,, \qquad e^{\frac{\Phi}{2}}g_{\tilde{m}y} = \frac{1}{e^{\frac{\hat{\Phi}}{2}}\hat{g}_{\hat{z}\hat{z}}}\hat{B}_{\hat{z}\tilde{m}}\,, \tag{3.10}$$

$$e^{\frac{\Phi}{2}}g_{\tilde{m}\tilde{n}} = e^{\frac{\hat{\Phi}}{2}}\hat{g}_{\tilde{m}\tilde{n}} + \frac{1}{e^{\frac{\hat{\Phi}}{2}}\hat{g}_{\hat{z}\hat{z}}}\left(\hat{B}_{\tilde{m}\hat{z}}\hat{B}_{\tilde{n}\hat{z}} - e^{\hat{\Phi}}\hat{g}_{\tilde{m}\hat{z}}\hat{g}_{\tilde{n}\hat{z}}\right)\,, \tag{3.11}$$

$$e^{2\Phi} = -\frac{e^{2\hat{\Phi}}}{e^{\frac{\hat{\Phi}}{2}}\hat{g}_{\hat{z}\hat{z}}}\,, \tag{3.12}$$

$$\begin{aligned} B_{\tilde{m}\tilde{n}} &= \hat{B}_{\tilde{m}\tilde{n}} + \frac{1}{\hat{g}_{\hat{z}\hat{z}}}\left(\hat{g}_{\tilde{m}\hat{z}}\hat{B}_{\tilde{n}\hat{z}} - \hat{g}_{\tilde{n}\hat{z}}\hat{B}_{\tilde{m}\hat{z}}\right)\,, \\ B_{y\tilde{m}} &= \frac{1}{\hat{g}_{\hat{z}\hat{z}}}\hat{g}_{\tilde{m}\hat{z}}\,. \end{aligned} \tag{3.13}$$

Clearly, the rules for NS-NS two-forms, eqs. (3.13), as well as the RR T-duality rules (3.5), (3.6), (3.7), (3.8) keep the same form in the Einstein frame.

4. T-duality rules for bosonic superforms

The T-duality rules for the bosonic superforms of type-IIA and type IIB supergravity, (2.1), (2.3), (2.9) and (2.2), (2.4), (2.10) can be derived from the study of the relations between the complete κ-symmetric super-Dp-brane actions, i.e. by the superfield generalization of the method proposed in [11]. We will describe these calculations in a longer paper. Here, instead, we will perform a straightforward superfield (more precisely, superform) generalization of the pure bosonic rules from [11], which reproduces exactly the same results. To this end

- one rewrites (3.10)–(3.13), (3.5)–(3.8) in differential form notations, using, in particular, the bosonic vielbein $e^a = dx^m e^a_m(x)$ instead of the metric $g_{mn}(x) = e^a_m e_{an}$.

Note that, e.g.,

$$\frac{\hat{g}_{\tilde{m}\hat{z}}}{\hat{g}_{\hat{z}\hat{z}}} \equiv \frac{\hat{e}_{\hat{a}\tilde{m}}\hat{e}_{\hat{z}}^{\hat{a}}}{\hat{e}_{\hat{b}\hat{z}}\hat{e}_{\hat{z}}^{\hat{b}}} = \frac{\hat{e}_{\tilde{m}}^{\#}}{\hat{e}_{\hat{z}}^{\#}}\,, \tag{4.1}$$

where the second equality is valid for the frame adapted to the isometry, i.e. follows from the usual Kaluza-Klein ansatz: $\hat{e}^{\tilde{a}} = d\tilde{x}^{\tilde{m}}\hat{e}_{\tilde{m}}^{\tilde{a}}(\tilde{x})$, ($\tilde{m} = 0, \ldots, 8$, $\tilde{a} = 0, \ldots, 8$), which implies $\hat{e}_{\hat{z}}^{\tilde{a}} = 0$ (cf. (2.26)).

- One replaces all fields by superfields, but assumes independence on one superspace bosonic coordinate, $\hat{z} = \hat{X}^9$ for type IIA and $y = X^9$ for type-IIB superfields, which describe the bosonic isometry directions.

In such a way, starting from eqs. (3.10)–(3.13), one reproduces the following *superform generalization of the NS-NS T-duality rules* (3.10)–(3.13)

$$\begin{aligned}
e^{\frac{\Phi(\tilde{Z})}{4}}E^{\tilde{a}(-)} &= e^{\frac{\hat{\Phi}(\tilde{Z})}{4}}\hat{E}^{\tilde{a}(-)}\,, && (4.2)\\
e^{\frac{\Phi}{4}}i_yE^{*} &= \frac{1}{e^{\frac{\hat{\Phi}}{4}}i_{\hat{z}}\hat{E}^{\#}}\,, && (4.3)\\
e^{\frac{\Phi}{4}}E^{*(-)} &= \frac{i_{\hat{z}}\hat{B}_2}{e^{\frac{\hat{\Phi}}{4}}i_{\hat{z}}\hat{E}^{\#}}\,, && (4.4)\\
e^{\Phi(\tilde{Z})} &= \frac{e^{\hat{\Phi}(\tilde{Z})}}{e^{\frac{\hat{\Phi}}{4}}i_{\hat{z}}\hat{E}^{\#}}\,, && (4.5)\\
i_yB_2 &= \frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}}\,, && (4.6)\\
B_2^{(-)} &= \hat{B}_2^{(-)} - i_{\hat{z}}\hat{B}_2 \wedge \frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}}\,. && (4.7)
\end{aligned}$$

Here we have used the decomposition (2.19) for type-IIA and type-IIB NS-NS superforms as well as the supervielbein forms adapted to the isometry, eqs. (2.24)–(2.29) (i.e., obeying the superfield Kaluza-Klein ansatz [26]).

In the same manner, writing eqs. (3.5)–(3.8) in the differential form notation and replacing all the forms by superforms subject to the superspace isometry conditions, one can arrive at the T-duality rules for the Ramond-Ramond superfield potentials,

$$\begin{aligned}
C^{(0)} &= i_{\hat{z}}\hat{C}_1\,, && (4.8)\\
i_yC_{2n} &= -\hat{C}_{2n-1}^{(-)} + \frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}} \wedge i_{\hat{z}}\hat{C}_{2n-1}\,, && (4.9)\\
C_{2n}^{(-)} &= i_{\hat{z}}\hat{C}_{2n+1} + i_{\hat{z}}\hat{B}_2 \wedge \left(\hat{C}_{2n-1}^{(-)} - \frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}} \wedge i_{\hat{z}}\hat{C}_{2n-1}\right)\,, && (4.10)\\
\text{for } n &= 1,2,3,4\,,\\
i_yC_{10} &= -\hat{C}_9^{(-)} + \frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}} \wedge i_{\hat{z}}\hat{C}_9\,. && (4.11)
\end{aligned}$$

Note that, if we had made use of supervielbein forms which were not adapted to the isometries as in (2.24)–(2.29), we would have arrived at more complicated expressions for the T-duality rules,

$$e^{\frac{\Phi(\tilde{Z})}{4}} E^{a(-)} = e^{\frac{\hat{\Phi}(\tilde{Z})}{4}} \hat{E}^{a(-)} + \left(e^{\frac{\hat{\Phi}(\tilde{Z})}{4}} \frac{\hat{E}^{a(-)} i_{\hat{z}} \hat{E}_a}{\hat{G}_{\hat{z}\hat{z}}} - e^{-\frac{\hat{\Phi}(\tilde{Z})}{4}} \frac{i_{\hat{z}} \hat{B}_2}{G_{\hat{z}\hat{z}}} \right) i_{\hat{z}} E^a \,, \tag{4.12}$$

$$e^{\frac{\Phi}{4}} i_y E^a = -\frac{i_{\hat{z}} \hat{E}^a}{\hat{G}_{\hat{z}\hat{z}}} \,, \qquad e^{2\Phi} = -\frac{e^{2\hat{\Phi}(\tilde{Z})}}{e^{\frac{\hat{\Phi}}{2}} \hat{G}_{\hat{z}\hat{z}}} \,, \qquad i_y B_2 = \frac{\hat{E}^{a(-)} i_{\hat{z}} \hat{E}_a}{\hat{G}_{\hat{z}\hat{z}}} \,, \tag{4.13}$$

$$B_2^{(-)} = \hat{B}_2^{(-)} - i_{\hat{z}} \hat{B}_2 \wedge \frac{\hat{E}^{a(-)} i_{\hat{z}} \hat{E}_a}{\hat{G}_{\hat{z}\hat{z}}} \,, \tag{4.14}$$

where

$$\hat{G}_{\hat{z}\hat{z}} \equiv i_{\hat{z}} \hat{E}^a i_{\hat{z}} \hat{E}_a \,. \tag{4.15}$$

These coincide with the rules from ref. [14] up to the dilaton factor, the discrepancy comes from the fact that ref. [14] deals with supervielbeins *in the string frame*

$$\begin{aligned} \text{type IIA}: \quad & (\hat{\mathcal{E}}^{\hat{a}}, \hat{\mathcal{E}}^{\alpha 1}, \hat{\mathcal{E}}^2_\alpha) = (e^{\frac{\hat{\Phi}}{4}} \hat{E}^{\hat{a}}, e^{\frac{\hat{\Phi}}{8}} \hat{E}^{\alpha 1}, e^{\frac{\hat{\Phi}}{8}} \hat{E}^2_\alpha) \,, \\ \text{type IIB}: \quad & (\mathcal{E}^{\hat{a}}, \mathcal{E}^{\alpha 1}, \mathcal{E}^{\alpha 2}) = (e^{\frac{\Phi}{4}} E^{\hat{a}}, e^{\frac{\Phi}{8}} E^{\alpha 1}, e^{\frac{\Phi}{8}} E^{\alpha 2}) \,. \end{aligned} \tag{4.16}$$

Below we show that the T-duality transformation rules for fermionic supervielbein forms can be derived by using the rules for the bosonic forms together with the supergravity constraints. These fermionic T-duality rules also coincide with the ones from [14] after transformation to the string frame.

However, our approach also allows one to derive the T-duality transformation rules for the RR superform potentials, eqs. (4.8)–(4.11). For general supervielbeins which are not adapted to the isometry they read

$$C^{(0)} = i_{\hat{z}} \hat{C}_1 \,, \tag{4.17}$$

$$i_y C_{2n} = -\hat{C}^{(-)}_{2n-1} + \frac{\hat{E}^{a(-)} i_{\hat{z}} \hat{E}_a}{\hat{G}_{\hat{z}\hat{z}}} \wedge i_{\hat{z}} \hat{C}_{2n-1} \,, \tag{4.18}$$

$$C^{(-)}_{2n} = i_{\hat{z}} \hat{C}_{2n+1} + i_{\hat{z}} \hat{B}_2 \wedge \left(\hat{C}^{(-)}_{2n-1} - \frac{\hat{E}^{a(-)} i_{\hat{z}} \hat{E}_a}{\hat{G}_{\hat{z}\hat{z}}} \wedge i_{\hat{z}} \hat{C}_{2n-1} \right) , \tag{4.19}$$

for $n = 1, 2, 3, 4$,

$$i_y C_{10} = -\hat{C}^{(-)}_9 + \frac{\hat{E}^{a(-)} i_{\hat{z}} \hat{E}_a}{\hat{G}_{\hat{z}\hat{z}}} \wedge i_{\hat{z}} \hat{C}_9 \,. \tag{4.20}$$

Eqs. (4.12)–(4.15), (4.17)–(4.19) possess manifest Lorentz ($SO(1,9)$) invariance. However, one shall keep in mind that they hold for superspaces with bosonic isometries. So we prefer to take advantage of supervielbein forms adapted to the isometry, eqs. (2.24)–(2.29), and to use a simpler and more geometrical form of the T-duality rules, eqs. (4.2)–(4.7), (4.8)–(4.10).[4]

4.1 Compact form of the T-duality rules for superforms

For future use it is convenient to present the T-duality rules (4.6), (4.7) as a relation between complete NS-NS superforms B_2 and $\hat{B}_2$. To this end one uses eqs. (2.19) to rewrite eq. (4.7) in the following form

$$B_2 = \hat{B}_2 - (dy + i_{\hat{z}}\hat{B}_2) \wedge \left(d\hat{z} + \frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}} \right) + dy \wedge d\hat{z} \, . \tag{4.21}$$

Furthermore, using the T-duality rules (4.3), (4.6) (which imply $E^{*(-)} \equiv E^* - i_y E^* dy = i_{\hat{z}}\hat{B}_2 i_y E^*$, i.e. $dy + i_{\hat{z}}\hat{B}_2 = E^*/i_y E^*$) and extracting $i_y E^* \, i_{\hat{z}}\hat{E}^{\#}$ in the common denominator, one finds

$$B_2 = \hat{B}_2 - \frac{1}{i_y E^*} E^* \wedge \hat{E}^{\#} \frac{1}{i_{\hat{z}}\hat{E}^{\#}} + dy \wedge d\hat{z} \, . \tag{4.22}$$

In the same manner the T-duality rules for RR superfield potentials, eqs. (4.8)–(4.10), can be collected in the following compact expression written in terms of the formal sums of all type-IIB and all type-IIA forms, (2.8) and (2.7),

$$C = i_{\hat{z}}\hat{C} + (dy + i_{\hat{z}}\hat{B}_2) \wedge \left(\hat{C}^{(-)} - \frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}} \wedge i_{\hat{z}}\hat{C} \right) . \tag{4.23}$$

eq. (4.23) can be used in its complete form to extract the rules for type-IIB $2n$ forms C_{2n} up to C_8. For C_{10} we have only i_y contraction of this equation (eq. (4.20)).

The relation inverse to (4.23) reads

$$\hat{C} = -i_y C + (d\hat{z} + i_y B_2) \wedge \left(C^{(-)} - i_y C \wedge \frac{E^{*(-)}}{i_y E^*} \right) \tag{4.24}$$

and can be used in its complete form for all type-IIA superforms, including $\hat{C}_9$.

5. T-duality rules for fermionic supervielbein forms

5.1 Supergravity constraints and spinorial cohomology approach

In this section we will show that T-duality rules for the fermionic supervielbein forms (2.30), (2.31) can be derived from the rules for the bosonic supervielbein (eqs. (4.2)–(4.5)) and

[4] If necessary, the generalization to supervielbein forms which are not adapted to the isometry can be made quite easily. To this end, one should take our equations and replace all the expressions with broken ten-dimensional Lorentz symmetry by formally covariant expressions, which are equal to the original ones for the adapted supervielbein forms. For instance,

$$\frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}} = \frac{\hat{E}^{\#(-)} i_{\hat{z}}\hat{E}^{\#}}{i_{\hat{z}}\hat{E}^{\#} i_{\hat{z}}\hat{E}^{\#}} = \frac{\hat{E}^{a(-)} i_{\hat{z}}\hat{E}_a}{\hat{G}_{\hat{z}\hat{z}}} \, , \qquad i_{\hat{z}}\hat{E}^{\#} \mapsto i_{\hat{z}}\hat{E}^{\#}\delta^{\hat{a}}_{\#} = i_{\hat{z}}\hat{E}^{\hat{a}} \mapsto i_{\hat{z}}\hat{E}^{a} \, ,$$

etc. Note that in this way one identifies $\hat{a} = a$ to make sense of relations like (4.12).

NS-NS superforms (eqs. (4.6), (4.7) or (4.22)) with the use of $D = 10$ type-IIA and type-IIB supergravity constraints [17].

These superspace constraints imply the following expression for the bosonic torsion 2-forms

$$\text{IIA} \qquad \hat{T}^{\hat{a}} := d\hat{E}^{\hat{a}} - \hat{E}^{\hat{b}} \wedge \hat{w}_{\hat{b}}{}^{\hat{a}} = -i\hat{E}^{\beta 1} \wedge \hat{E}^{\gamma 1}\sigma^{\hat{a}}_{\beta\gamma} - i\hat{E}^2_\beta \wedge \hat{E}^2_\gamma \tilde{\sigma}^{\hat{a}\beta\gamma} \,, \tag{5.1}$$

$$\text{IIB} \qquad T^a := dE^a - E^b \wedge w_b{}^a = -iE^{\beta 1} \wedge E^{\gamma 1}\sigma^a_{\beta\gamma} - iE^{\beta 2} \wedge E^{\gamma 2}\sigma^a_{\beta\gamma} \,, \tag{5.2}$$

and for the NS-NS gauge superfield strength

$$\begin{aligned} \text{type IIA} \qquad \hat{H}_3 := d\hat{B}_2 = & -ie^{\frac{1}{2}\hat{\Phi}}\hat{E}^{\hat{a}} \wedge (\hat{E}^{\beta 1} \wedge \hat{E}^{\gamma 1}\sigma_{\hat{a}\beta\gamma} - \hat{E}^2_\beta \wedge \hat{E}^2_\gamma \tilde{\sigma}_{\hat{a}}^{\beta\gamma}) + \\ & + \frac{e^{\frac{1}{2}\hat{\Phi}}}{4}\hat{E}^{\hat{b}} \wedge \hat{E}^{\hat{a}} \wedge (\hat{E}^{\beta 1}\sigma_{\hat{a}\hat{b}\beta}{}^\gamma \hat{\nabla}_{\gamma 1}\hat{\Phi} + \hat{E}^2_\gamma \sigma_{\hat{a}\hat{b}\beta}{}^\gamma \hat{\nabla}^\beta_2 \hat{\Phi}) + \\ & + \frac{1}{3!}\hat{E}^{\hat{c}} \wedge \hat{E}^{\hat{b}} \wedge \hat{E}^{\hat{a}} \hat{H}_{\hat{a}\hat{b}\hat{c}} \,, \end{aligned} \tag{5.3}$$

$$\begin{aligned} \text{type IIB} : H_3 = dB_2 \; = & \; -ie^{\frac{1}{2}\Phi}E^a \wedge (E^{\alpha 1} \wedge E^{\beta 1} - E^{\alpha 2} \wedge E^{\beta 2})\sigma_{a\ \alpha\beta} + \\ & + \frac{1}{4}e^{\frac{1}{2}\Phi}E^b \wedge E^a \wedge \left(E^{\alpha 1}\nabla_{\beta 1}\Phi - E^{\alpha 2}\nabla_{\beta 2}\Phi\right) (\sigma_{ab})_\alpha{}^\beta + \\ & + \frac{1}{3!}E^c \wedge E^b \wedge E^a H_{abc} \,, \end{aligned} \tag{5.4}$$

as well as certain expressions for the fermionic torsion 2-forms ($\hat{T}^{\beta 1} := \mathcal{D}\hat{E}^{\beta 1}$, $\hat{T}^2_\beta := \mathcal{D}\hat{E}^2_\beta$ and $T^{\beta 1,2} := \mathcal{D}E^{\beta 1,2}$), curvatures of the spin connections ($\hat{R}^{\hat{a}\hat{b}} := d\hat{w}^{\hat{a}\hat{b}} - \hat{w}^{\hat{a}\hat{c}} \wedge \hat{w}_{\hat{c}}{}^{\hat{b}}$ and $R^{ab} := dw^{ab} - w^{ac} \wedge w_c{}^b$) and field strengths of the RR superforms ($\hat{R}_{2n+2} = d\hat{C}_{2n+1} - \hat{C}_{2n-1} \wedge \hat{H}_3$ and $R_{2n+1} := dC_{2n} - C_{2n-2} \wedge H_3$, see below). For shortness we will call *'constraints'* all the above mentioned relations (although they include not only the proper constraints, see [27, 19, 20], but also their consequences).

The complete set of the superfield T-duality rules should be consistent with all the constraints; i.e. it should map the complete set of the type-IIA constraints, eqs. (5.1), (5.3), etc., into the complete set of the type-IIB constraints, eqs. (5.2), (5.4), etc. Clearly, a part of such correspondence should be sufficient (and, indeed, is sufficient) to derive the T-duality rules for the fermionic superforms, while the check of the correspondence between the remaining constraints promises to be the quite involved exercise.

Fortunately, the situation can be simplified drastically by the use of theorems about interdependence of the constraints (see, e.g., [27, 19, 20]). The differential form constraints contain a number of spin-tensor and tensor relations (e.g. $H_{\alpha 1\, \beta 1\, \gamma 1} = 0$, $H_{\alpha 1\, \beta 1\, \gamma 2} = 0$, $\ldots$, $H_{\alpha 1\, \beta 1\, c} = -2ie^{\frac{\Phi}{2}}\sigma_{c\alpha\beta}$, $\ldots$, $H_{\alpha 1\, b\, c} = \frac{1}{2}e^{\frac{\Phi}{2}}(\sigma_{bc})_\alpha{}^\beta \nabla_{\beta 1}\Phi$, $\ldots$ in (5.4)) which we call *'components'* (not to be confused with components of superfields). It is convenient to classify them by dimension (in energy units and corresponding to the lower case indices, e.g. 3/2, 3/2, ..., 2, ..., 5/2, ... in the above example). Then, using the Bianchi identities, $d\hat{H}_3 \equiv 0$ and $dH_3 \equiv 0$, one finds [19, 20] that all the components of dimension more than 2 in the constraints (5.3) and (5.4) can be derived from the lower dimensional components of the same equations (i.e. coefficients for a basic 3-forms $E^A \wedge E^B \wedge E^C$ with not more than one bosonic supervielbein form E^a) and the constraints for the bosonic torsion two-forms, (5.1) and (5.2), respectively.[5]

This allows us to search for the T-duality rules for the fermionic supervielbein forms by requiring the consistency of the rules for the bosonic superforms (eqs. (4.2)–(4.11)) with the lower dimensional components of eqs. (5.3), (5.4) and the (complete) constraints (5.1), (5.2). It is convenient to organize this procedure as follows. We will study the consistency of the T-duality rules with the complete constraints (5.1)–(5.4), but ignoring in (5.3), (5.4) the terms $\mathcal{O}(E^a \wedge E^b)$ and $\mathcal{O}(\hat{E}^{\hat{a}} \wedge \hat{E}^{\hat{b}})$, which include more than one bosonic supervielbein form. Such method is close in spirit to the 'spinorial cohomology approach' developed recently in [28] (for a different problem, see also [29]).

Due to the same reason we can also omit from the consideration all the expressions for the fermionic torsions, $\hat{T}^{\alpha 1}$, $\hat{T}^2_\alpha$ and $T^{\alpha 1,2}$, and for the curvatures, $\hat{R}^{\hat{a}\hat{b}} := d\hat{w}^{\hat{a}\hat{b}} - \hat{w}^{\hat{a}\hat{c}} \wedge \hat{w}_{\hat{c}}{}^{\hat{b}}$ and $R^{ab} := dw^{ab} - w^{ac} \wedge w_c{}^b$. Their form can be derived from eqs. (5.1)–(5.4) with the use of Bianchi identities (see footnote 5) and, hence, their consistency with the T-duality rules should be guarantied by the consistency of eqs. (5.1)–(5.4). The consistency of the T-duality rules with the constraints for RR field strength will be discussed in section 5.

5.2 Torsion constraints and superfield Kaluza-Klein ansatz

First observe that using the superfield Kaluza-Klein ansatz for the bosonic supervielbein forms, eqs. (2.26)–(2.29), in the torsion constraints (5.1) with $\hat{a} = \#$ and (5.2) with $a = *$, one finds (in our notation $\sigma^\# = -\sigma_\# \equiv \sigma^9 \equiv \sigma^* = -\sigma_*$)

$$\text{type IIA} \qquad i_{\hat{z}} \hat{E}^{\alpha 1} = \frac{i}{2} \tilde{\sigma}^{\# \alpha\beta} \hat{\nabla}_{\beta 1} i_{\hat{z}} \hat{E}^\# , \qquad i_{\hat{z}} \hat{E}^2_\alpha = \frac{i}{2} \sigma^\#_{\alpha\beta} \hat{\nabla}^\beta_2 i_{\hat{z}} \hat{E}^\# , \tag{5.5}$$

$$\text{type IIB} \qquad i_y E^{\alpha 1} = \frac{i}{2} \tilde{\sigma}^{\# \alpha\beta} \nabla_{\beta 1} i_y E^* , \qquad i_y E^{\alpha 2} = \frac{i}{2} \tilde{\sigma}^{\# \alpha\beta} \nabla_{\beta 2} i_y E^* , \tag{5.6}$$

as well as

$$\hat{w}_{\hat{z}\tilde{a}}{}^\# = -\hat{\nabla}_a i_{\hat{z}} \hat{E}^\# , \qquad w_{y\tilde{a}}{}^* = -\nabla_a i_y E^* . \tag{5.7}$$

Eqs. (5.5), (5.6) imply that our problem reduces essentially to the search for the T-duality rules for $\hat{E}^{\alpha 1[-]}$, $\hat{E}^{2[-]}_\alpha$, and $E^{\alpha 1,2[-]}$ which, then, will allow to define the rules for the fermionic covariant derivative entering eqs. (5.5), (5.6) (see below).

[5] Actually, in such calculations one needs to know as well the constraints for the fermionic torsion 2-forms. However, they, as well as the expressions for the curvatures of spin connections, can be completely restored from the constraints for the bosonic torsion two-forms, (5.1) and (5.2), with the use of Bianchi identities

$$\mathcal{D}\hat{T}^{\hat{a}} = -\hat{E}^{\hat{b}} \wedge \hat{R}_{\hat{b}}{}^{\hat{a}} , \qquad \mathcal{D}\hat{T}^{\alpha 1} = -\hat{E}^{\beta 1} \wedge \hat{R}^{\hat{b}\hat{a}} \frac{1}{4} \sigma_{\hat{b}\hat{a}\beta}{}^\alpha , \qquad \mathcal{D}\hat{T}^2_\beta = \hat{E}^2_\beta \wedge \hat{R}^{\hat{b}\hat{a}} \frac{1}{4} \sigma_{\hat{b}\hat{a}\alpha}{}^\beta , \qquad \mathcal{D}\hat{R}^{\hat{b}\hat{a}} \equiv 0 ,$$

and

$$\mathcal{D}T^a = -E^b \wedge R_b{}^a , \qquad \mathcal{D}T^{\alpha 1} = -E^{\beta 1} \wedge R^{ba} \frac{1}{4} \sigma_{ba\beta}{}^\alpha , \qquad \mathcal{D}T^{\alpha 2} = -E^{\beta 2} \wedge R^{ba} \frac{1}{4} \sigma_{ba\beta}{}^\alpha , \qquad \mathcal{D}R^{ba} \equiv 0 .$$

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Eq. (5.1) with $\hat{a} = \tilde{a} = 0, \ldots, 8$ and (5.2) with $a = \tilde{a}$ give the expressions for type IIA and type-IIB representations for the bosonic torsion of nine-dimensional superspace (2.13),

$$\begin{aligned} \text{type IIA}: \quad \hat{\mathcal{D}}^{[-]}\hat{E}^{\tilde{a}(-)} &:= d\hat{E}^{\tilde{a}(-)} - \hat{E}^{\tilde{b}(-)} \wedge \hat{w}^{[-]}{}_{\tilde{b}}{}^{\tilde{a}} = \\ &= -i\hat{E}^{\beta 1[-]} \wedge \hat{E}^{\gamma 1[-]}\sigma^{\tilde{a}}_{\beta\gamma} - i\hat{E}^{2[-]}_{\beta} \wedge \hat{E}^{2[-]}_{\gamma}\, \tilde{\sigma}^{\tilde{a}\beta\gamma}, \qquad (5.8) \\ \text{type IIB}: \quad \mathcal{D}^{[-]}E^{\tilde{a}(-)} &:= dE^{\tilde{a}(-)} - E^{\tilde{b}(-)} \wedge w^{[-]}{}_{\tilde{b}}{}^{\tilde{a}} = \\ &= -iE^{\beta 1[-]} \wedge E^{\gamma 1[-]}\sigma^{\tilde{a}}_{\beta\gamma} - iE^{\beta 2[-]} \wedge E^{\gamma 2[-]}\sigma^{\tilde{a}}_{\beta\gamma}, \qquad (5.9) \end{aligned}$$

as well as (in the parts proportional to $\hat{E}^{\#}$ and E^{*}) specify completely the parts $\hat{w}^{\#\tilde{a}[-]}$ and $w^{*\tilde{a}[-]}$ of the spin connections. Collecting the latter result with eqs. (5.7), we find

$$\begin{aligned} \text{type IIA}: \quad \hat{w}_{\#}{}^{\tilde{a}} &= \frac{1}{i_{\hat{z}}\hat{E}^{\#}}(\hat{E}^{\tilde{b}(-)}\hat{w}_{\hat{z}\tilde{b}}{}^{\tilde{a}} - 2i\hat{E}^{\alpha 1[-]}\sigma^{\tilde{a}}_{\alpha\beta}i_{\hat{z}}\hat{E}^{\beta 1} - \\ &\quad - 2i\hat{E}^{2[-]}_{\alpha}\tilde{\sigma}^{\tilde{a}\alpha\beta}i_{\hat{z}}\hat{E}^{2[-]}_{\beta} - \hat{E}^{\#}\hat{\nabla}^{\tilde{a}}i_{\hat{z}}\hat{E}^{\#}), \qquad (5.10) \\ \text{type IIB}: \quad w_{*}{}^{\tilde{a}} &= \frac{1}{i_y\hat{E}^{*}}(E^{\tilde{b}(-)}w_{y\tilde{b}}{}^{\tilde{a}} - 2iE^{\alpha 1[-]}\sigma^{\tilde{a}}_{\alpha\beta}i_yE^{\beta 1} - \\ &\quad - 2iE^{\alpha 2[-]}\sigma^{\tilde{a}}_{\alpha\beta}i_yE^{\beta 2} - E^{*}\nabla^{\tilde{a}}i_y\hat{E}^{*}). \qquad (5.11) \end{aligned}$$

5.3 T-duality rules for fermionic forms from the supergravity constraints I. General structure from NS-NS constraints

First, let us observe for future use that eq. (4.21) allows to derive the T-duality rule for the NS-NS gauge superfield strength (5.3) and (5.4). It is convenient to present them in the form

$$H_3 = \hat{H}_3 - \left(d\hat{z} + \frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}}\right) \wedge i_{\hat{z}}\hat{H}_3 + \left(dy + \frac{E^{*(-)}}{i_y\hat{E}^{*}}\right) \wedge i_yH_3, \qquad (5.12)$$

using the identities $di_{\hat{z}}\hat{B}_2 = -i_{\hat{z}}\hat{H}_3$, $di_yB_2 = -i_yH_3$ implied by the isometry conditions.

On the other hand, taking the exterior derivative of eq. (4.22) and using the definition of the superspace torsion, eqs. (5.1) and (5.2), one obtains another (equivalent, but more convenient) form of the T-duality rules (5.12),

$$\begin{aligned} H_3 = \hat{H}_3 &- \frac{1}{i_yE^{*}\,i_{\hat{z}}\hat{E}^{\#}}\,E^{*} \wedge \hat{T}^{\#} + \frac{1}{i_yE^{*}\,i_{\hat{z}}\hat{E}^{\#}}\,\hat{E}^{\#} \wedge T^{*} + \qquad (5.13) \\ &+ \frac{1}{i_yE^{*}\,i_{\hat{z}}\hat{E}^{\#}}\left(\hat{E}^{\#} \wedge \mathcal{E}^{\tilde{b}} \wedge w_{\tilde{b}}{}^{*} - E^{*} \wedge E^{\tilde{b}} \wedge \hat{w}_{\tilde{b}}{}^{\#} + E^{*} \wedge \hat{E}^{\#} \wedge d\,log|i_yE^{*}\,i_{\hat{z}}\hat{E}^{\#}|\right). \end{aligned}$$

Substituting (5.1)–(5.4) into (5.13) and taking into account eqs. (4.2)–(4.5) one finds after straightforward algebraic manipulations

$$\begin{aligned} &-ie^{\frac{1}{2}\hat{\Phi}}E^{\tilde{a}(-)} \wedge [(E^{\beta 1} \wedge E^{\gamma 1}\sigma_{\tilde{a}\beta\gamma} - e^{\frac{1}{4}(\hat{\Phi}-\Phi)}\,\hat{E}^{\beta 1} \wedge \hat{E}^{\gamma 1}\sigma_{\tilde{a}\beta\gamma}) - \\ &\qquad - (E^{\beta 2} \wedge E^{\gamma 2}\sigma_{\tilde{a}\beta\gamma} + e^{\frac{1}{4}(\hat{\Phi}-\Phi)}\,\hat{E}^{2}_{\beta} \wedge \hat{E}^{2}_{\gamma}\tilde{\sigma}^{\beta\gamma}_{\tilde{a}})] + \\ &+ ie^{\frac{1}{2}\Phi}(E^{*} + e^{\frac{1}{4}(\hat{\Phi}-\Phi)}\,\hat{E}^{\#}) \wedge [E^{\beta 1} \wedge E^{\gamma 1}\sigma^{\#}_{\beta\gamma} - e^{\frac{1}{4}(\hat{\Phi}-\Phi)}\,\hat{E}^{\beta 1} \wedge \hat{E}^{\gamma 1}\sigma^{\#}_{\beta\gamma}] - \\ &- ie^{\frac{1}{2}\Phi}(E^{*} - e^{\frac{1}{4}(\hat{\Phi}-\Phi)}\,\hat{E}^{\#}) \wedge [E^{\beta 2} \wedge E^{\gamma 2}\sigma^{\#}_{\beta\gamma} + e^{\frac{1}{4}(\hat{\Phi}-\Phi)}\,\hat{E}^{2}_{\beta} \wedge \hat{E}^{2}_{\gamma}\tilde{\sigma}^{\#\beta\gamma}] = \\ &\qquad = \mathcal{O}(E^{a} \wedge E^{b},\ E^{a} \wedge \hat{E}^{\#}). \qquad (5.14) \end{aligned}$$

In eq. (5.14) $\mathcal{O}(E^a \wedge E^b\,,\; E^a \wedge \hat{E}^{\#})$ denotes the terms containing at least two bosonic supervielbein forms of the *underlying superspace* $\mathcal{M}^{(11|32)}$ (2.15). As only 11 bosonic supervielbein forms, e.g. (2.17), can be considered as independent on $\mathcal{M}^{(11|32)}$, eq. (5.14) implies

$$\begin{aligned} E^{\beta 1} \wedge E^{\gamma 1} \sigma_{\tilde{a}\beta\gamma} - E^{\beta 2} \wedge E^{\gamma 2} \sigma_{\tilde{a}\beta\gamma} &= e^{\frac{1}{4}(\hat{\Phi}-\Phi)}\, \hat{E}^{\beta 1} \wedge \hat{E}^{\gamma 1} \sigma_{\tilde{a}\beta\gamma} \\ &\quad - e^{\frac{1}{4}(\hat{\Phi}-\Phi)}\, \hat{E}^{2}_{\beta} \wedge \hat{E}^{2}_{\gamma} \tilde{\sigma}_{\tilde{a}}^{\beta\gamma} + \mathcal{O}(E^a\,,\; \hat{E}^{\#})\,, \end{aligned} \tag{5.15}$$

$$E^{\beta 1} \wedge E^{\gamma 1} \sigma^{\#}_{\beta\gamma} - e^{\frac{1}{4}(\hat{\Phi}-\Phi)}\, \hat{E}^{\beta 1} \wedge \hat{E}^{\gamma 1} \sigma^{\#}_{\beta\gamma} = \mathcal{O}(E^a\,,\; \hat{E}^{\#})\,, \tag{5.16}$$

$$E^{\beta 2} \wedge E^{\gamma 2} \sigma^{\#}_{\beta\gamma} + e^{\frac{1}{4}(\hat{\Phi}-\Phi)}\, \hat{E}^{2}_{\beta} \wedge \hat{E}^{2}_{\gamma} \tilde{\sigma}^{\#\beta\gamma} = \mathcal{O}(E^a\,,\; \hat{E}^{\#})\,. \tag{5.17}$$

Hence eqs. (5.15)–(5.17) suggest the following relation between the type-IIA and type-IIB fermionic supervielbein forms

$$e^{\frac{1}{8}\Phi} E^{\beta 1[-]} = e^{\frac{1}{8}\hat{\Phi}}\, (\hat{E}^{\beta 1[-]} + \hat{E}^{\tilde{a}(-)} \nu_{\tilde{a}}^{\beta 1})\,, \tag{5.18}$$

$$e^{\frac{1}{8}\Phi} E^{\beta 2[-]} = e^{\frac{1}{8}\hat{\Phi}}\, \tilde{\sigma}^{\#\beta\gamma}\, (\hat{E}^{2[-]}_{\gamma} + \hat{E}^{\tilde{a}(-)} \nu_{\tilde{a}\gamma}{}^{2})\,, \tag{5.19}$$

where $\nu_{\tilde{a}}^{\beta 1}$ and $\nu_{\tilde{a}\gamma}{}^{2}$ are indefinite coefficients. Below we will find their explicit form from the torsion constraints (5.1) and (5.2).

In conclusion of this section we note that eqs. (5.18), (5.19) are sufficient to find the relation between covariant spinor derivatives acting on a scalar superfield $V(\tilde{Z})$ defined on the nine-dimensional superspace $\mathcal{M}^{(9|32)}$ (2.13). The differential acting on such a superfield $V(\tilde{Z})$, $dV(\tilde{Z}) = d\tilde{Z}^{\tilde{M}} \partial_{\tilde{M}} V(\tilde{Z})$, can be decomposed either on type-IIA or on type-IIB supervielbein forms,

$$\begin{aligned} dV(\tilde{Z}) &= d\tilde{Z}^{\tilde{M}} \partial_{\tilde{M}} V(\tilde{Z}) = \hat{E}^{\tilde{a}(-)} \hat{\nabla}_{\tilde{a}} V(\tilde{Z}) + \hat{E}^{\alpha 1[-]} \hat{\nabla}_{\alpha 1} V(\tilde{Z}) + \hat{E}^{2[-]}_{\alpha} \hat{\nabla}^{\alpha}_{2} V(\tilde{Z}) \\ &= E^{\tilde{a}(-)} \nabla_{\tilde{a}} V(\tilde{Z}) + E^{\alpha 1[-]} \nabla_{\alpha 1} V(\tilde{Z}) + E^{\alpha 2[-]} \nabla_{\alpha 2} V(\tilde{Z})\,. \end{aligned} \tag{5.20}$$

Substituting the T-duality rules (4.2), (5.18), (5.19), one finds that eq. (5.20) implies, in particular,

$$e^{-\frac{1}{8}\Phi} \nabla_{\alpha 1} V(\tilde{Z}) = e^{-\frac{1}{8}\hat{\Phi}} \hat{\nabla}_{\alpha 1} V(\tilde{Z})\,, \qquad e^{-\frac{1}{8}\Phi} \nabla_{\alpha 2} V(\tilde{Z}) = -e^{-\frac{1}{8}\hat{\Phi}} \sigma^{\#}_{\alpha\beta} \hat{\nabla}^{\beta}_{2} V(\tilde{Z})\,. \tag{5.21}$$

5.4 T-duality rules for fermionic forms from the supergravity constraints II. Complete form from torsion constraints

First, let us observe that the T-duality rule (4.2) implies the following relation between the type-IIA and type-IIB representations for the torsion of nine-dimensional superspace $\mathcal{M}^{(9|32)}$ (2.13) (see l.h.s's of eqs. (5.8) and (5.9))

$$e^{\frac{\Phi}{4}} \mathcal{D}^{[-]} E^{\tilde{a}(-)} = e^{\frac{\hat{\Phi}}{4}} \left[\hat{\mathcal{D}}^{[-]} \hat{E}^{\tilde{a}(-)} + \frac{1}{4} \hat{E}^{\tilde{a}(-)} \wedge d(\hat{\Phi} - \Phi) + \hat{E}^{\tilde{b}(-)} \wedge \left(\hat{w}^{[-]}{}_{\tilde{b}}{}^{\tilde{a}} - w^{[-]}{}_{\tilde{b}}{}^{\tilde{a}} \right) \right]. \tag{5.22}$$

Using the constraints (5.8) and (5.9) and the relations between fermionic forms (5.18), (5.19) one finds that the lower dimensional components of the eq. (5.22) are satisfied identically, while the dimension 3/2 components provide one with the equations for $\nu_{\tilde{a}}^{\beta 1}$ and $\nu_{\tilde{a}\gamma}^{\ 2}$,

$$2i\tilde{\sigma}^{\tilde{a}}_{\alpha\beta}\nu^{\beta 1}_{\tilde{a}} = \frac{1}{4}\delta_{\tilde{b}}{}^{\tilde{a}}\left(\hat{\nabla}_{\alpha 1}\hat{\Phi} - e^{\frac{1}{8}(\hat{\Phi}-\Phi)}\nabla_{\alpha 1}\Phi\right) + \Delta\hat{w}^{[-]}{}_{\alpha 1\tilde{b}}{}^{\tilde{a}}\,, \tag{5.23}$$

$$2i\tilde{\sigma}^{\tilde{a}\alpha\beta}\nu_{\tilde{b}\gamma}^{\ 2} = \frac{1}{4}\delta_{\tilde{b}}{}^{\tilde{a}}\left(\hat{\nabla}_{2}^{\alpha}\hat{\Phi} - e^{\frac{1}{8}(\hat{\Phi}-\Phi)}\tilde{\sigma}^{\#\alpha\beta}\nabla_{\beta 2}\Phi\right) + \Delta\hat{w}^{[-]}{}_{2}^{\alpha}{}_{\tilde{b}}{}^{\tilde{a}}\,, \tag{5.24}$$

where $\Delta\hat{w}^{[-]}{}_{\tilde{b}}{}^{\tilde{a}} \equiv (\hat{w}^{[-]} - w^{[-]})_{\tilde{b}}{}^{\tilde{a}} = \hat{E}^{\tilde{c}(-)}\Delta\hat{w}^{[-]}{}_{\tilde{c}\tilde{b}}{}^{\tilde{a}} + \hat{E}^{\alpha 1[-]}\Delta\hat{w}^{[-]}{}_{\alpha 1\tilde{b}}{}^{\tilde{a}} + \hat{E}^{2[-]}_{\alpha}\Delta\hat{w}^{[-]}{}_{2}^{\alpha}{}_{\tilde{b}}{}^{\tilde{a}}$. The solutions of these equations,

$$\nu_{\tilde{a}}^{\beta 1} = -\frac{i}{8}\tilde{\sigma}^{\alpha\beta}_{\tilde{a}}\left(\hat{\nabla}_{\alpha 1}\hat{\Phi} - e^{\frac{1}{8}(\hat{\Phi}-\Phi)}\nabla_{\alpha 1}\Phi\right), \tag{5.25}$$

$$\nu_{\tilde{a}\alpha}^{\ 2} = -\frac{i}{8}\sigma_{\tilde{a}\alpha\beta}\left(\hat{\nabla}_{2}^{\alpha}\hat{\Phi} - e^{\frac{1}{8}(\hat{\Phi}-\Phi)}\tilde{\sigma}^{\#\alpha\beta}\nabla_{\beta 2}\Phi\right), \tag{5.26}$$

provide us with the following final form of the essential fermionic T-duality rules

$$e^{\frac{1}{8}\Phi}(E^{\beta 1[-]} - \frac{i}{8}E^{\tilde{a}(-)}\tilde{\sigma}_{\tilde{a}}{}^{\beta\gamma}\nabla_{\gamma 1}\Phi) = e^{\frac{1}{8}\hat{\Phi}}\left(\hat{E}^{\beta 1[-]} - \frac{i}{8}\hat{E}^{\tilde{a}(-)}\tilde{\sigma}_{\tilde{a}}{}^{\beta\gamma}\hat{\nabla}_{\gamma 1}\hat{\Phi}\right), \tag{5.27}$$

$$e^{\frac{1}{8}\Phi}(E^{\beta 2[-]} - \frac{i}{8}E^{\tilde{a}(-)}\tilde{\sigma}_{\tilde{a}}{}^{\beta\gamma}\nabla_{\gamma 2}\Phi) = e^{\frac{1}{8}\hat{\Phi}}\,\tilde{\sigma}^{\#\beta\gamma}\left(\hat{E}^{2[-]}_{\gamma} - \frac{i}{8}\hat{E}^{\tilde{a}(-)}\sigma_{\tilde{a}\beta\gamma}\hat{\nabla}_{2}^{\gamma}\hat{\Phi}\right). \tag{5.28}$$

Note that if we used (5.20) to decompose $d\Phi$ in eq. (5.22) on the type-IIA superforms, eqs. (5.25), (5.26) would read

$$\nu_{\tilde{a}}^{\beta 1} = -\frac{i}{8}\tilde{\sigma}^{\alpha\beta}_{\tilde{a}}\hat{\nabla}_{\alpha 1}\left(\hat{\Phi} - \Phi\right), \tag{5.29}$$

$$\nu_{\tilde{a}\alpha}^{\ 2} = -\frac{i}{8}\sigma_{\tilde{a}\alpha\beta}\hat{\nabla}_{2}^{\alpha}\left(\hat{\Phi} - \Phi\right), \tag{5.30}$$

where $(\hat{\Phi} - \Phi)$ could be expressed through the type-IIA superfields by using the T-duality rule (4.5),

$$\hat{\Phi}(\tilde{Z}) - \Phi(\tilde{Z}) = \ln\left(e^{\frac{1}{4}\hat{\Phi}}i_{\hat{z}}\hat{E}^{\#}\right). \tag{5.31}$$

Such notation allows us to rewrite the rules (5.27), (5.28) in slightly different form, see eqs. (5.39), (5.40) below. As a by-product, on these stages one also obtains the T-duality rules for the 'nine-dimensional' part of the spin connections (see (2.34), (2.37))

$$\begin{aligned} w^{\tilde{b}\tilde{a}[-]} = {} & \hat{w}^{\tilde{b}\tilde{a}[-]} + \frac{1}{4}\hat{E}^{\alpha 1[-]}(\sigma^{\tilde{b}\tilde{a}})_{\alpha}{}^{\beta}\hat{\nabla}_{\beta 1}(\hat{\Phi} - \Phi) - \frac{1}{4}\hat{E}^{2[-]}_{\alpha}(\sigma^{\tilde{b}\tilde{a}})_{\beta}{}^{\alpha}\hat{\nabla}_{2}^{\beta}(\hat{\Phi} - \Phi) + \\ & + E^{[\tilde{b}(-)}\hat{\nabla}^{\tilde{a}]}(\hat{\Phi} - \Phi) - \frac{3i}{64}E^{(-)}_{\tilde{c}}\Big(\tilde{\sigma}^{\tilde{c}\tilde{b}\tilde{a}\alpha\beta}\hat{\nabla}_{\alpha 1}(\hat{\Phi} - \Phi)\,\hat{\nabla}_{\beta 1}(\hat{\Phi} - \Phi) + \\ & + \sigma^{\tilde{c}\tilde{b}\tilde{a}}{}_{\alpha\beta}\hat{\nabla}_{2}^{\alpha}(\hat{\Phi} - \Phi)\,\hat{\nabla}_{2}^{\beta}(\hat{\Phi} - \Phi)\Big), \end{aligned} \tag{5.32}$$

where one can substitute (5.31) for $(\hat{\Phi} - \Phi)$.

To complete the fermionic T-duality rules we have to find the relation between the fermionic superfields $i_{\hat{z}}\hat{E}^{\alpha 1}$, $i_{\hat{z}}\hat{E}^{2}_{\alpha}$ and $i_{y}E^{\alpha 1,2}$. As that are composed, eqs. (5.5), (5.6), to this end it is sufficient to use the relation between covariant spinor derivatives, eqs. (5.21), for $V(\tilde{Z}) = \hat{E}^{\#}_{\hat{z}}(\tilde{Z}) \equiv i_{\hat{z}}\hat{E}^{\#}$ and $V(\tilde{Z}) = \hat{E}^{*}_{y}(\tilde{Z}) \equiv i_{y}\hat{E}^{*}$. In such a way one finds the following T-duality rules for the fermionic superfields $E^{\alpha 1}_{\hat{z}}(\tilde{Z}) \equiv i_{\hat{z}}\hat{E}^{\alpha 1}$, $\hat{E}_{\hat{z}}{}^{2}_{\alpha}(\tilde{Z}) \equiv i_{\hat{z}}\hat{E}^{2}_{\alpha}$ and $E^{\alpha 1,2}_{y}(\tilde{Z}) \equiv i_{y}E^{\alpha 1,2}$:

$$e^{-\frac{1}{8}\Phi}\left(\frac{i_y E^{\beta 1}}{i_y E^{*}} + \frac{i}{8}\tilde{\sigma}^{*\beta\gamma}\nabla_{\gamma 1}\Phi\right) = -\, e^{-\frac{1}{8}\hat{\Phi}}\,\left(\frac{i_{\hat{z}}\hat{E}^{\beta 1}}{i_{\hat{z}}\hat{E}^{\#}} + \frac{i}{8}\tilde{\sigma}^{\#\beta\gamma}\hat{\nabla}_{\gamma 1}\hat{\Phi}\right), \tag{5.33}$$

$$e^{-\frac{1}{8}\Phi}\left(\frac{i_y E^{\beta 2}}{i_y E^{*}} + \frac{i}{8}\tilde{\sigma}^{*\beta\gamma}\nabla_{\gamma 2}\Phi\right) = e^{-\frac{1}{8}\hat{\Phi}}\,\tilde{\sigma}^{\#\beta\gamma}\,\left(\frac{i_{\hat{z}}\hat{E}^{2}_{\gamma}}{i_{\hat{z}}\hat{E}^{\#}} + \frac{i}{8}\sigma^{\#}_{\beta\gamma}\hat{\nabla}^{\gamma}_{2}\hat{\Phi}\right). \tag{5.34}$$

Eqs. (5.27), (5.28) can be collected together with eqs. (5.33), (5.34) in the following fermionic T-duality rules involving only the *complete* forms and their contractions

$$\begin{aligned} e^{\frac{1}{8}\Phi}(E^{\beta 1} - \frac{i}{8}E^{a}\tilde{\sigma}_{a}{}^{\beta\gamma}\nabla_{\gamma 1}\Phi) &= e^{\frac{1}{8}\hat{\Phi}}\,(\hat{E}^{\beta 1} - \frac{i}{8}\hat{E}^{a}\tilde{\sigma}_{a}{}^{\beta\gamma}\hat{\nabla}_{\gamma 1}\hat{\Phi}) - \\ &\quad - e^{\frac{1}{8}\hat{\Phi}}\,\left(\hat{E}^{\#} + e^{\frac{1}{4}(\Phi-\hat{\Phi})}E^{*}\right)\left(\frac{i_{\hat{z}}\hat{E}^{\beta 1}}{i_{\hat{z}}\hat{E}^{\#}} + \frac{i}{8}\tilde{\sigma}^{\#\beta\gamma}\hat{\nabla}_{\gamma 1}\hat{\Phi}\right), \end{aligned} \tag{5.35}$$

$$\begin{aligned} e^{\frac{1}{8}\Phi}(E^{\beta 2} - \frac{i}{8}E^{a}\tilde{\sigma}_{a}{}^{\beta\gamma}\nabla_{\gamma 2}\Phi) &= e^{\frac{1}{8}\hat{\Phi}}\,\tilde{\sigma}^{\#\beta\gamma}\,(\hat{E}^{2}_{\gamma} - \frac{i}{8}\hat{E}^{a}\sigma_{a\beta\gamma}\hat{\nabla}^{\gamma}_{2}\hat{\Phi}) - \\ &\quad - e^{\frac{1}{8}\hat{\Phi}}\,\left(\hat{E}^{\#} - e^{\frac{1}{4}(\Phi-\hat{\Phi})}E^{*}\right)\tilde{\sigma}^{\#\beta\gamma}\left(\frac{i_{\hat{z}}\hat{E}^{2}_{\gamma}}{i_{\hat{z}}\hat{E}^{\#}} + \frac{i}{8}\sigma^{\#}_{\beta\gamma}\hat{\nabla}^{\gamma}_{2}\hat{\Phi}\right). \end{aligned} \tag{5.36}$$

Indeed, due to the last terms in (5.35), (5.36), the relations obtained by contractions of these equations with $i_{\hat{z}}$ are satisfied identically, while the contractions of these equations with i_y reproduce the T-duality rules for spinor superfields (5.33), (5.34). The parts of eqs. (5.35), (5.36) which do not contain neither $\hat{E}^{\#}$ nor E^{*} reproduce eqs. (5.27), (5.28).

It might be useful to rewrite the fermionic T-duality rules in the more standard form similar to the one of eqs. (4.2)–(4.7). To this end one uses (5.21) and (4.5) to present eqs. (5.33), (5.34) and (5.27), (5.28) as

$$e^{-\frac{1}{8}\Phi}\frac{i_y E^{\beta 1}}{i_y E^{*}} = -\, e^{-\frac{1}{8}\hat{\Phi}}\,\left(\frac{i_{\hat{z}}\hat{E}^{\beta 1}}{i_{\hat{z}}\hat{E}^{\#}} + \frac{i}{4}\tilde{\sigma}^{\#\beta\gamma}\hat{\nabla}_{\gamma 1}\hat{\Phi} - \frac{i}{8}\tilde{\sigma}^{\#\beta\gamma}\hat{\nabla}_{\gamma 1}\ln\left(e^{\frac{\hat{\Phi}}{4}}i_{\hat{z}}E^{\#}\right)\right), \tag{5.37}$$

$$e^{-\frac{1}{8}\Phi}\frac{i_y E^{\beta 2}}{i_y E^{*}} = e^{-\frac{1}{8}\hat{\Phi}}\,\tilde{\sigma}^{\#\beta\gamma}\,\left(\frac{i_{\hat{z}}\hat{E}^{2}_{\gamma}}{i_{\hat{z}}\hat{E}^{\#}} + \frac{i}{4}\sigma^{\#}_{\beta\gamma}\hat{\nabla}^{\gamma}_{2}\hat{\Phi} - \frac{i}{8}\sigma^{\#}_{\beta\gamma}\hat{\nabla}^{\gamma}_{2}\ln\left(e^{\frac{\hat{\Phi}}{4}}i_{\hat{z}}E^{\#}\right)\right) \tag{5.38}$$

and

$$e^{\frac{1}{8}\Phi}E^{\beta 1[-]} = e^{\frac{1}{8}\hat{\Phi}}\,\left(\hat{E}^{\beta 1[-]} - \frac{i}{8}\hat{E}^{\tilde{a}(-)}\tilde{\sigma}_{\tilde{a}}{}^{\beta\gamma}\hat{\nabla}_{\gamma 1}\ln\left(e^{\frac{\hat{\Phi}}{4}}i_{\hat{z}}E^{\#}\right)\right), \tag{5.39}$$

$$e^{\frac{1}{8}\Phi}E^{\beta 2[-]} = e^{\frac{1}{8}\hat{\Phi}}\,\tilde{\sigma}^{\#\beta\gamma}\,\left(\hat{E}^{2[-]}_{\gamma} - \frac{i}{8}\hat{E}^{\tilde{a}(-)}\sigma_{\tilde{a}\beta\gamma}\hat{\nabla}^{\gamma}_{2}\ln\left(e^{\frac{\hat{\Phi}}{4}}i_{\hat{z}}E^{\#}\right)\right). \tag{5.40}$$

Let us comment on the relation of the above results with the T-duality rules for fermionic superforms *in the string frame* (see eq. (4.16)) derived in ref. [14]. To this end, at first, one ignores the dilaton superfield in eqs. (5.33), (5.34) and, at second, one passes to the general supervielbein forms (not adapted to the isometries). The result is

$$\mathcal{E}_y^{\beta 1} = \frac{\hat{\mathcal{E}}_{\hat{z}}^{\beta 1}}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\,, \qquad \mathcal{E}_y^{\beta 2} = \frac{\hat{\mathcal{E}}_{\hat{z}}^{a}\tilde{\sigma}_a^{\beta\gamma}\hat{\mathcal{E}}_{\hat{z}\gamma}^{2}}{\sqrt{|\hat{\mathcal{G}}_{\hat{z}\hat{z}}|}\,\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\,, \tag{5.41}$$

where

$$\hat{\mathcal{G}}_{\hat{z}\hat{z}} \equiv \hat{\mathcal{E}}_{\hat{z}}^{\hat{a}}\hat{\mathcal{E}}_{\hat{z}\hat{a}}\,. \tag{5.42}$$

In the same manner, ignoring the inputs from dilaton superfields in eqs. (5.27), (5.28), and passing form $E^{[-]}$ to $E^{(-)}$ by the use of (2.32), (2.33), we arrive at

$$\begin{aligned}
\mathcal{E}^{\beta 1(-)} &= \hat{\mathcal{E}}^{\beta 1(-)} - \frac{\hat{\mathcal{E}}^{a(-)} i_{\hat{z}}\hat{\mathcal{E}}_a}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\, i_{\hat{z}}\hat{\mathcal{E}}^{\beta 1} + \frac{i_{\hat{z}}\hat{B}_2}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\, i_{\hat{z}}\hat{\mathcal{E}}^{\beta 1}\,, \\
\mathcal{E}^{\beta 2(-)} &= -\frac{i_{\hat{z}}\hat{\mathcal{E}}^{\hat{a}}\tilde{\sigma}_{\hat{a}}^{\beta\gamma}}{\sqrt{|\hat{\mathcal{G}}_{\hat{z}\hat{z}}|}} \left(\hat{\mathcal{E}}_{\gamma}^{2(-)} - \frac{\hat{\mathcal{E}}^{a(-)} i_{\hat{z}}\hat{\mathcal{E}}_a}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\, i_{\hat{z}}\hat{\mathcal{E}}_{\gamma}^{2(-)} - \frac{i_{\hat{z}}\hat{B}_2}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\, i_{\hat{z}}\hat{\mathcal{E}}_{\gamma}^{2}\right).
\end{aligned} \tag{5.43}$$

Eqs. (5.41), (5.43) coincide with the T-duality rules for the fermionic supervielbein forms presented in ref. [14].[6]

6. Consistency of the T-duality rules with the constraints for RR field strengths

Thus all the T-duality rules are restored with the use of the torsion constraints and the constraints for the field strengths of the NS-NS gauge superforms. The question remains: whether these the T-duality rules, including eqs. (5.35), (5.36) and (4.23), are consistent with the constraints for the field strengths

$$R = dC - C \wedge H_3 = R_1 \oplus R_3 \oplus R_5 \oplus R_7 \oplus R_9\,, \tag{6.1}$$

$$\hat{R} = d\hat{C} - \hat{C} \wedge \hat{H}_3 = \hat{R}_2 \oplus \hat{R}_4 \oplus \hat{R}_6 \oplus \hat{R}_8 \oplus \hat{R}_{10}\,, \tag{6.2}$$

of the RR superforms (2.8), (2.10) and (2.7), (2.9). These constraints can be found in [17]. For our consideration it is convenient to collect them in the following equations for the formal sums of the RR field strengths

$$\hat{R} = d\hat{C} - \hat{C} \wedge \hat{H}_3 = 2ie^{-\frac{3}{4}\hat{\Phi}}\, \hat{E}^{\alpha 1} \wedge \hat{E}_{\beta}^{2} \wedge \hat{\bar{\gamma}}(\hat{\Phi})^{\beta}{}_{\alpha} + \cdots\,, \tag{6.3}$$

$$R = dC - C \wedge H_3 = 2ie^{-\frac{3}{4}\Phi}\, E^{\alpha 2} \wedge E^{\beta 1} \wedge \bar{\sigma}(\Phi)_{\alpha\beta} + \cdots\,. \tag{6.4}$$

[6] An evident factor $1/\sqrt{|\hat{\mathcal{G}}_{\hat{z}\hat{z}}|}$ in eqs. (5.41), (5.43) for $\mathcal{E}^{\beta 2}$ should be restored in eqs. of ref. [14].

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Here the matrix valued formal sums of differential (super)forms $\hat{\bar{\gamma}}(\hat{\Phi})^{\beta}{}_{\alpha}$, $\bar{\sigma}(\Phi)_{\alpha\beta}$ are defined by

$$\hat{\bar{\gamma}}(\hat{\Phi})^{\beta}{}_{\alpha} \equiv \delta^{\beta}{}_{\alpha} \oplus \hat{\bar{\gamma}}^{(2)}(\hat{\Phi})^{\beta}{}_{\alpha} \oplus \hat{\bar{\gamma}}^{(4)}(\hat{\Phi})^{\beta}{}_{\alpha} \oplus \hat{\bar{\gamma}}^{(6)}(\hat{\Phi})^{\beta}{}_{\alpha} \oplus \hat{\bar{\gamma}}^{(8)}(\hat{\Phi})^{\beta}{}_{\alpha} \tag{6.5}$$

$$\hat{\bar{\gamma}}^{(2n)}(\hat{\Phi})^{\beta}{}_{\alpha} = \frac{e^{\frac{n}{2}\hat{\Phi}}}{(2n)!}\hat{E}^{a_{2n}} \wedge \cdots \wedge \hat{E}^{a_1}\sigma_{a_1 \ldots a_{2n}}{}^{\beta}{}_{\alpha}\,, \tag{6.6}$$

$$\bar{\sigma}(\Phi)_{\alpha\beta} = \bar{\sigma}(\Phi)^{(1)}_{\alpha\beta} \oplus \bar{\sigma}(\Phi)^{(3)}_{\alpha\beta} \oplus \bar{\sigma}(\Phi)^{(5)}_{\alpha\beta} \oplus \bar{\sigma}(\Phi)^{(7)}_{\alpha\beta}\,, \tag{6.7}$$

$$\bar{\sigma}(\Phi)^{(2n+1)}_{\alpha\beta} \equiv \frac{e^{\frac{2n+1}{4}\Phi}}{(2n+1)!}E^{\hat{a}_{2n+1}} \wedge \cdots \wedge E^{\hat{a}_1}\sigma_{\hat{a}_1 \ldots \hat{a}_{2n+1}\alpha\beta}\,. \tag{6.8}$$

The terms denoted by ellipses in eqs. (6.3), (6.4) include *not more than one* fermionic supervielbein form and can be ignored due to the following reasons. When the expressions for certain differential q-forms ($q > 2$) are extracted from (6.3), (6.4), the terms with less than two fermionic supervielbein forms contain more bosonic supervielbein forms and, thus, describe the higher dimensional components of the q-form equation. However, as in the case of NS-NS superfield strengths (see section 5.1), such higher dimensional components can be derived as a consequence of the lowest dimensional ones and the torsion constraints (5.1), (5.2) with the use of superspace Bianchi identities (which can be collected in $d\hat{R} \equiv \hat{R} \wedge \hat{H}_3$ and $dR \equiv R \wedge H_3$). Thus we can conventionally ignore them in the analysis of T-duality as there consistency is guarantied provided the lowest dimensional equations are consistent with the T-duality rules. This is a 'bottom-up' form of the spinor cohomology approach of section 5.1.[7]

Having in hands the explicit T-duality rules for the RR fields, eq. (4.23), one can obtain by direct calculations the T-duality rules for their field strengths. To this end one can

- take the derivative of eq. (4.23);
- use the conditions of isometry for NS-NS two forms and RR forms

$$d(i_y B_2) = -i_y H_3\,, \quad d(i_{\hat{z}}\hat{B}_2) = -i_{\hat{z}}\hat{H}_3\,, \quad d(i_y C) = -i_y(dC)\,, \quad d(i_{\hat{z}}\hat{C}) = -i_{\hat{z}}(d\hat{C})\,,$$

  to arrive at the expression in terms of the field strength;
- use eqs. (6.4), (6.3) to obtain the expressions in terms of generalized field strength R and $\hat{R}$ instead of dC, $d\hat{C}$;
- observe that in the result of such calculations all the terms involving potential(s) C (after the use of eq. (4.23)) can be collected in the expression

$$C \wedge \left(\hat{H}_3 - H_3 + \left(dy + \frac{E^{*(-)}}{i_y E^*}\right) \wedge i_y H_3 - \left(d\hat{z} + \frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}}\right) \wedge i_{\hat{z}}\hat{H}_3\right)$$

  which vanishes in accordance with (5.12).

[7]The constraint $R_1 = dC_0 = e^{-\Phi}E^{\underline{\alpha}1}\nabla_{\underline{\alpha}2}\Phi - e^{-\Phi}E^{\underline{\alpha}2}\nabla_{\underline{\alpha}1}\Phi + E^{\underline{a}}R_{\underline{a}}$ for the axion 'field strength', which is completely hidden in ellipses in eq. (6.4), also can be derived from the torsion constraints.

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In such a way we arrive at the T-duality rules for the RR field strengths (6.4), (6.3),

$$R = -i_{\hat{z}}\hat{R} + \left(dy + \frac{E^{*(-)}}{i_y E^*} \right) \wedge \left(\hat{R} + \left(d\hat{z} + \frac{\hat{E}^{\#(-)}}{i_{\hat{z}}\hat{E}^{\#}} \right) \wedge i_{\hat{z}}\hat{R} \right), \tag{6.9}$$

which are gauge invariant and resemble the rules (4.23) for the RR superfield potentials.

Now one can verify that the derived T-duality rules are completely consistent with lower dimensional spin-tensor relations involved into the differential form constraints (6.3), (6.4) for RR field strength. To this end one i) substitutes the constraints (6.3), (6.4) into (6.9) ii) checks that the resulting equation is satisfied identically when the T-duality rules for NS-NS superfields, eq. (4.2)–(4.7), and for the fermionic forms, eqs. (5.35), (5.36), are taken into account. In the light of above consideration (on the 'bottom-up' form of the spinor cohomology approach), on this way one can ignore the terms denoted by ellipses in (6.3) and (6.4), as well as the terms with less than two fermionic forms which appear after substitution of the fermionic T-duality rules into eqs. (6.3), (6.4). With this shortcut the explicit check of the consistency reduces to a simple exercise in sigma-matrix algebra, which we leave for reader.

7. Conclusion

In this paper we have obtained the complete set of the *superfield T-duality rules* which are summarized in the appendix A (see sections A.1 and A.2). These are the relations between all superfield potentials of type-IIA and type-IIB supergravity, including fermionic supervielbein forms and all the Ramond-Ramond superfield potentials. For their derivation we used the supervielbeins in the Einstein frame which are adapted to the isometries (i.e. obey the superfield generalization of the Kaluza-Klein ansatz [26]). We also present the rules formulated for general supervielbeins in the string frame, where our results for NS-NS superfields and fermionic superforms coincide with the ones obtained in [14] (appendix A.2). Thus we completed the rules from [14] by the T-duality rules for RR gauge superfields. We also propose the differential form representation for the T-duality rules (appendix A.3) which have allowed to verify their consistency with the complete set of supergravity constraints.

Let us stress the basic observation which has allowed us to sort out the problem that hampered the way to the complete superfield generalization of the T-duality rules. It consists in the possibility to treat the T-duality as a transformation of supervielbein and other superforms rather than of the superspace coordinates. This implies the identification of all the fermionic coordinates of the *curved* type-IIA and type-IIB superspaces, as well as of all but one their bosonic coordinates. The identification of the fermionic coordinates of curved type-IIA and type-IIB superspaces is possible due to the fact that fermionic coordinates of a curved superspace carry neither spinor indices nor chiralities, but rather are transformed by superspace diffeomorphisms. The different chiralities of the fermionic coordinate of the *flat* type-IIA and type-IIB superspaces originate in different chiralities of the fermionic supervielbein forms of the *curved* superspaces and can be reproduced in the flat superspace limit, after the fermionic supervielbein forms are identified with

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the exterior derivatives of the fermionic coordinates. As far as the curved superspaces are concerned, assuming that the type IIA and type-IIB supergravities have one bosonic isometry direction, $\hat{z}$ and y respectively, one may consider them as defined on surfaces $y = 0$ and $\hat{z} = 0$ in the underlying eleven-dimensional superspace $\mathcal{M}^{(11|32)}$ (2.15). The intersection of these surfaces gives a nine-dimensional superspace $\mathcal{M}^{(9|32)}$ (2.13).

Our results clarify the relation of T-duality with superfield formulations of supergravity and, as we hope, might provide new insights in M-theory. Our approach can be also extended to the more complicated $\mathrm{SO}(n,n)$ T-duality provided the superfield generalization of the Kaluza-Klein ansatz for the dimensional reduction down to $d = 10 - n$ dimensions is elaborated for these cases.

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A. Summary of superfield T-duality rules

A.1 T-duality rules with supervielbein in the Einstein frame adapted to the isometry (2.24)–(2.29)

For the bosonic supervielbeins adapted to the isometries, i.e. obeying Kaluza-Klein ansatz [26]

$$\text{type IIA}: \hat{E}^{\hat{a}} = (\hat{E}^{\tilde{a}}, \hat{E}^{\#}), \qquad \hat{E}^{\tilde{a}} = \hat{E}^{\tilde{a}(-)} = d\tilde{Z}^M \hat{E}_M^{\tilde{a}}(\tilde{Z}) \tag{A.1}$$

$$\text{type IIB}: E^{a} = (E^{\tilde{a}}, E^{*}), \qquad E^{\tilde{a}} = E^{\tilde{a}(-)} = d\tilde{Z}^M E_M^{\tilde{a}}(\tilde{Z}), \tag{A.2}$$

the T-duality rules for NS-NS superfields have the form: for the bosonic supervielbeins

$$e^{\frac{\Phi(\tilde{Z})}{4}} E^{\tilde{a}(-)} = e^{\frac{\hat{\Phi}(\tilde{Z})}{4}} \hat{E}^{\tilde{a}(-)}, \qquad e^{\frac{\Phi}{4}} E^{*}_{y} = \frac{1}{e^{\frac{\hat{\Phi}}{4}} \hat{E}^{\#}_{\hat{z}}}, \qquad e^{\frac{\Phi}{4}} E^{*(-)} = \frac{i_{\hat{z}} \hat{B}_2}{e^{\frac{\hat{\Phi}}{4}} \hat{E}^{\#}_{\hat{z}}}, \tag{A.3}$$

for dilaton

$$e^{\Phi(\tilde{Z})} = \frac{e^{\hat{\Phi}(\tilde{Z})}}{e^{\frac{\hat{\Phi}}{4}} \hat{E}^{\#}_{\hat{z}}}, \tag{A.4}$$

and for the NS-NS superforms

$$i_y B_2 = \frac{\hat{E}^{\#(-)}}{\hat{E}^{\#}_{\hat{z}}}, \qquad B_2^{(-)} = \hat{B}_2^{(-)} - i_{\hat{z}} \hat{B}_2 \wedge \frac{\hat{E}^{\#(-)}}{\hat{E}^{\#}_{\hat{z}}}. \tag{A.5}$$

The T-duality rules for fermionic supervielbeins are

$$e^{-\frac{1}{8}\Phi}\frac{E_y^{\beta 1}}{E_y^*} = -\,e^{-\frac{1}{8}\hat{\Phi}}\left(\frac{\hat{E}_{\hat{z}}^{\beta 1}}{\hat{E}_{\hat{z}}^{\#}} + \frac{i}{4}\tilde{\sigma}^{\#\beta\gamma}\hat{\nabla}_{\gamma 1}\hat{\Phi} - \frac{i}{8}\tilde{\sigma}^{\#\beta\gamma}\hat{\nabla}_{\gamma 1}\ln\left(e^{\frac{\hat{\Phi}}{4}}E_{\hat{z}}^{\#}\right)\right), \tag{A.6}$$

$$e^{-\frac{1}{8}\Phi}\frac{E_y^{\beta 2}}{E_y^*} = e^{-\frac{1}{8}\hat{\Phi}}\,\tilde{\sigma}^{\#\beta\gamma}\left(\frac{\hat{E}_{\hat{z}\gamma}^{\ 2}}{\hat{E}_{\hat{z}}^{\#}} + \frac{i}{4}\sigma^{\#}_{\beta\gamma}\hat{\nabla}_2^{\gamma}\hat{\Phi} - \frac{i}{8}\sigma^{\#}_{\beta\gamma}\hat{\nabla}_2^{\gamma}\ln\left(e^{\frac{\hat{\Phi}}{4}}E_{\hat{z}}^{\#}\right)\right), \tag{A.7}$$

$$e^{\frac{1}{8}\Phi}E^{\beta 1[-]} = e^{\frac{1}{8}\hat{\Phi}}\left(\hat{E}^{\beta 1[-]} - \frac{i}{8}\hat{E}^{\tilde{a}(-)}\tilde{\sigma}_{\tilde{a}}^{\ \beta\gamma}\hat{\nabla}_{\gamma 1}\ln\left(e^{\frac{\hat{\Phi}}{4}}E_{\hat{z}}^{\#}\right)\right), \tag{A.8}$$

$$e^{\frac{1}{8}\Phi}E^{\beta 2[-]} = e^{\frac{1}{8}\hat{\Phi}}\,\tilde{\sigma}^{\#\beta\gamma}\left(\hat{E}_{\gamma}^{2[-]} - \frac{i}{8}\hat{E}^{\tilde{a}(-)}\sigma_{\tilde{a}\beta\gamma}\hat{\nabla}_2^{\gamma}\ln\left(e^{\frac{\hat{\Phi}}{4}}E_{\hat{z}}^{\#}\right)\right). \tag{A.9}$$

T-duality rules for the RR superform potentials are

$$\begin{aligned}
C^{(0)} &= i_{\hat{z}}\hat{C}_1 \equiv \hat{C}_{\hat{z}}^{(1)}\,,\\
i_y C_{2n} &= -\hat{C}_{2n-1}^{(-)} + \frac{\hat{E}^{\#(-)}}{\hat{E}_{\hat{z}}^{\#}}\wedge i_{\hat{z}}\hat{C}_{2n-1}\,, \quad n = 1,2,3,4,5\,,\\
C_{2n}^{(-)} &= i_{\hat{z}}\hat{C}_{2n+1} + i_{\hat{z}}\hat{B}_2\wedge\left(\hat{C}_{2n-1}^{(-)} - \frac{\hat{E}^{\#(-)}}{\hat{E}_{\hat{z}}^{\#}}\wedge i_{\hat{z}}\hat{C}_{2n-1}\right),\\
n &= 1,2,3,4\,.
\end{aligned} \tag{A.10}$$

A.2 T-duality rules with supervielbeins in the string frame which are not adapted to the isometry

The relation between Einstein frame and string frame supervielbein is given in eq. (4.16). In the string frame the T-duality rules for NS-NS superfields acquire the form (the supervielbein forms are not assumed to be adapted to the isometries as in appendix A.1)

$$\mathcal{E}^{a(-)} = \hat{\mathcal{E}}^{a(-)} + \frac{\hat{\mathcal{E}}^{a(-)}\hat{\mathcal{E}}_{\hat{z}\,a}}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}} - \frac{i_{\hat{z}}\hat{B}_2}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\mathcal{E}_{\hat{z}}^{a}\,, \qquad i_y\mathcal{E}^a = -\frac{\hat{\mathcal{E}}_{\hat{z}}^{a}}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\,, \tag{A.11}$$

$$e^{2\Phi} = -\frac{e^{2\hat{\Phi}(\tilde{Z})}}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\,, \tag{A.12}$$

$$B_2^{(-)} = \hat{B}_2^{(-)} - i_{\hat{z}}\hat{B}_2\wedge\frac{\hat{\mathcal{E}}^{a(-)}\hat{\mathcal{E}}_{\hat{z}a}}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\,, \qquad i_y B_2 = \frac{\hat{\mathcal{E}}^{a(-)}\hat{\mathcal{E}}_{\hat{z}a}}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\,. \tag{A.13}$$

The T-duality rules for the fermionic supervielbeins are

$$\begin{aligned}
\mathcal{E}_y^{\beta 1} &= \frac{\hat{\mathcal{E}}_{\hat{z}}^{\beta 1}}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\,, \qquad \mathcal{E}_y^{\beta 2} = \frac{\hat{\mathcal{E}}_{\hat{z}}^{a}\tilde{\sigma}_a^{\beta\gamma}\hat{\mathcal{E}}_{\hat{z}\gamma}^{\ 2}}{\sqrt{|\hat{\mathcal{G}}_{\hat{z}\hat{z}}|}\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\,,\\
\mathcal{E}^{\beta 1(-)} &= \hat{\mathcal{E}}^{\beta 1(-)} - \frac{\hat{\mathcal{E}}^{a(-)}\hat{\mathcal{E}}_{\hat{z}\,a}}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\hat{\mathcal{E}}_{\hat{z}}^{\beta 1} + \frac{i_{\hat{z}}\hat{B}_2}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\hat{\mathcal{E}}_{\hat{z}}^{\beta 1}\,,\\
\mathcal{E}^{\beta 2(-)} &= -\frac{\hat{\mathcal{E}}_{\hat{z}}^{\hat{a}}\tilde{\sigma}_a^{\beta\gamma}}{\sqrt{|\hat{\mathcal{G}}_{\hat{z}\hat{z}}|}}\left(\hat{\mathcal{E}}_{\gamma}^{2(-)} - \frac{\hat{\mathcal{E}}^{a(-)}\hat{\mathcal{E}}_{\hat{z}a}}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\hat{\mathcal{E}}_{\hat{z}\gamma}^{\ 2} - \frac{i_{\hat{z}}\hat{B}_2}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}}\hat{\mathcal{E}}_{\hat{z}\gamma}^{\ 2}\right).
\end{aligned} \tag{A.14}$$

The T-duality rules for the RR-superform potentials are

$$C^{(0)} = i_{\hat{z}}\hat{C}_1 \equiv \hat{C}^{(1)}_{\hat{z}}\,, \tag{A.15}$$

$$i_y C_{2n} = -\hat{C}^{(-)}_{2n-1} + \frac{\hat{\mathcal{E}}^{a(-)}\hat{\mathcal{E}}_{\hat{z}a}}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}} \wedge i_{\hat{z}}\hat{C}_{2n-1}\,, \quad n = 1,2,3,4,5\,, \tag{A.16}$$

$$C^{(-)}_{2n} = i_{\hat{z}}\hat{C}_{2n+1} + i_{\hat{z}}\hat{B}_2 \wedge \left(\hat{C}^{(-)}_{2n-1} - \frac{\hat{\mathcal{E}}^{a(-)}\hat{\mathcal{E}}_{\hat{z}a}}{\hat{\mathcal{G}}_{\hat{z}\hat{z}}} \wedge i_{\hat{z}}\hat{C}_{2n-1}\right), \tag{A.17}$$

$$n = 1,2,3,4\,.$$

$$i_y C_{10} = -\hat{C}^{(-)}_9 + \frac{\hat{\mathcal{E}}^{a(-)} i_{\hat{z}}\hat{\mathcal{E}}_a}{\hat{G}_{\hat{z}\hat{z}}} \wedge i_{\hat{z}}\hat{C}_9\,. \tag{A.18}$$

Here

$$\hat{\mathcal{G}}_{\hat{z}\hat{z}} \equiv \hat{\mathcal{E}}^{\hat{a}}_{\hat{z}}\hat{\mathcal{E}}_{\hat{z}\hat{a}}\,. \tag{A.19}$$

A.3 The complete differential form representation for the T-duality rules (A.5)–(A.10)

For NS-NS superforms:

$$B_2 = \hat{B}_2 - (dy + i_{\hat{z}}\hat{B}_2) \wedge \left(d\hat{z} + \frac{\hat{E}^{\#(-)}}{\hat{E}^{\#}_{\hat{z}}}\right) + dy \wedge d\hat{z}\,, \tag{A.20}$$

or

$$B_2 = \hat{B}_2 - \frac{1}{E^*_y}\, E^* \wedge \hat{E}^{\#}\, \frac{1}{\hat{E}^{\#}_{\hat{z}}} + dy \wedge d\hat{z}\,. \tag{A.21}$$

For RR-superforms:

$$C = i_{\hat{z}}\hat{C} + (dy + i_{\hat{z}}\hat{B}_2) \wedge \left(\hat{C}^{(-)} - \frac{\hat{E}^{\#(-)}}{\hat{E}^{\#}_{\hat{z}}} \wedge i_{\hat{z}}\hat{C}\right), \tag{A.22}$$

or

$$\hat{C} = -i_y C + (d\hat{z} + i_y B_2) \wedge \left(C^{(-)} - i_y C \wedge \frac{E^{*(-)}}{E^*_y}\right). \tag{A.23}$$

For fermionic supervielbeins:

$$\begin{aligned} e^{\frac{1}{8}\Phi}(E^{\beta 1} - \frac{i}{8}E^a\tilde{\sigma}_a{}^{\beta\gamma}\nabla_{\gamma 1}\Phi) &= e^{\frac{1}{8}\hat{\Phi}}\,(\hat{E}^{\beta 1} - \frac{i}{8}\hat{E}^a\tilde{\sigma}_a{}^{\beta\gamma}\hat{\nabla}_{\gamma 1}\hat{\Phi}) - \\ &- e^{\frac{1}{8}\hat{\Phi}}\,\left(\hat{E}^{\#} + e^{\frac{1}{4}(\Phi-\hat{\Phi})}E^*\right)\left(\frac{\hat{E}^{\beta 1}_{\hat{z}}}{\hat{E}^{\#}_{\hat{z}}} + \frac{i}{8}\tilde{\sigma}^{\#\beta\gamma}\hat{\nabla}_{\gamma 1}\hat{\Phi}\right), \end{aligned} \tag{A.24}$$

$$\begin{aligned} e^{\frac{1}{8}\Phi}(E^{\beta 2} - \frac{i}{8}E^a\tilde{\sigma}_a{}^{\beta\gamma}\nabla_{\gamma 2}\Phi) &= e^{\frac{1}{8}\hat{\Phi}}\,\tilde{\sigma}^{\#\beta\gamma}\,(\hat{E}^2_{\gamma} - \frac{i}{8}\hat{E}^a\sigma_{a\beta\gamma}\hat{\nabla}^{\gamma}_2\hat{\Phi}) - \\ &- e^{\frac{1}{8}\hat{\Phi}}\,\left(\hat{E}^{\#} - e^{\frac{1}{4}(\Phi-\hat{\Phi})}E^*\right)\tilde{\sigma}^{\#\beta\gamma}\left(\frac{\hat{E}_{\hat{z}}{}^2_{\gamma}}{\hat{E}^{\#}_{\hat{z}}} + \frac{i}{8}\sigma^{\#}_{\beta\gamma}\hat{\nabla}^{\gamma}_2\hat{\Phi}\right). \end{aligned} \tag{A.25}$$

A.4 T-duality for the field strengths

For NS-NS superfield strengths $\hat{H}_3 = d\hat{B}_2$ and $H_3 = dB_2$ the T-duality rules read

$$H_3 = \hat{H}_3 - \left(d\hat{z} + \frac{\hat{E}^{\#(-)}}{\hat{E}^{\#}_{\hat{z}}} \right) \wedge i_{\hat{z}} \hat{H}_3 \; + \left(dy + \frac{E^{*(-)}}{E^{*}_{y}} \right) \wedge i_y H_3 \,, \tag{A.26}$$

or, equivalently,

$$\begin{aligned} H_3 \, &= \hat{H}_3 - \frac{1}{E^*_y \, \hat{E}^{\#}_{\hat{z}}} \, E^* \wedge \hat{T}^{\#} + \frac{1}{E^*_y \, \hat{E}^{\#}_{\hat{z}}} \, \hat{E}^{\#} \wedge T^* + \\ &+ \frac{1}{E^*_y \, \hat{E}^{\#}_{\hat{z}}} \left(\hat{E}^{\#} \wedge \mathcal{E}^{\tilde{b}} \wedge w_{\tilde{b}}{}^{*} - E^* \wedge E^{\tilde{b}} \wedge \hat{w}_{\tilde{b}}{}^{\#} + E^* \wedge \hat{E}^{\#} \wedge d \, \log |E^*_y \, \hat{E}^{\#}_{\hat{z}}| \right) \end{aligned} \tag{A.27}$$

For RR superfield strengths the T-duality rules can be collected in the relation

$$R = -i_{\hat{z}} \hat{R} + \left(dy + \frac{E^{*(-)}}{E^{*}_{y}} \right) \wedge \left(\hat{R} + \left(d\hat{z} + \frac{\hat{E}^{\#(-)}}{\hat{E}^{\#}_{\hat{z}}} \right) \wedge i_{\hat{z}} \hat{R} \right) . \tag{A.28}$$

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